5.4/5.6

Seperable Differential Equations

Ex #1 $f'(x) = \frac{1}{x} + 2x$, $P(1,-1)$.

$f(x) = \ln|x| + x^2 + C$

$-1 = \ln 1 + 1 + C$

$-1 = 1 + C$

$-2 = C$

$\boxed{f(x) = \ln|x| + x^2 - 2}$

Ex #2 $y' = \frac{x}{y}$ Find the general solution.

$\frac{dy}{dx} = \frac{x}{y}$

$y\,dy = x\,dx$

$\boxed{\frac{y^2}{2} = \frac{x^2}{2} + C}$

Ex #3 Solve for the general solution if $\frac{dy}{dx} = (xy)^2$. Then identify the particular solution if the curve passes the point $P(1,1)$.

$\frac{dy}{dx} = x^2y^2$

$\frac{dy}{y^2} = x^2\,dx$

★ $-\frac{1}{y} = \frac{x^3}{3} + C$

$-1 = \frac{1}{3} + C$

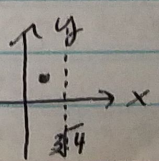
★ $-\frac{4}{3} = C$

$-\frac{1}{y} = \frac{x^3}{3} - \frac{4}{3}$

$-\frac{1}{y} = \frac{x^3 - 4}{3}$

$\frac{1}{y} = \frac{4 - x^3}{3}$

$\boxed{y = \frac{3}{4 - x^3}, \quad x < \sqrt[3]{4}}$



The Law of Exponential Exchange

Rate of change is directly proportional to amount present.

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{y} = k\,dt$$

$$\ln|y| = kt + C$$

$$|y| = e^{kt+C}$$

$$|y| = e^{C}e^{kt}$$

$$y = \pm e^{C}e^{kt} \qquad y(0) = y_0$$

$$y_0 = \pm e^{C}e^{0}$$

$$y_0 = \pm e^{C}$$

$$y = y_0 e^{kt}$$

EX #4 A population is growing at a rate proportional to the population. On May 1st, the population is 10 & grows to 50 in 30 days. How many days will it take for the pop. to reach 100?

$$\frac{dy}{dt} = ky \rightarrow y = y_0 e^{kt}$$

$$50 = 10e^{30k}$$

$$5 = e^{30k}$$

$$\ln 5 = 30k$$

$$\frac{1}{30}\ln 5 = k$$

$$y = 10e^{\frac{t}{30}\ln 5} = 10e^{\ln 5^{t/30}}$$

$$y = 10(5)^{t/30}$$

$$100 = 10(5)^{t/30}$$

$$30\log_5 10 = t$$

$$t \approx 43 \text{ days after May 1st!}$$

(11)

Ex#5 (#1194) You are driving along a highway at a steady 60mph (88 ft/sec) when you see an accident ahead & slam on the breaks. What constant deceleration is req. to stop your car in 242 ft?

want: $a = -k$

know: $v_0 = 88$ ft/sec $\quad v_f = 0$ ft/sec

$x_0 = 0$ ft

$x_f = 242$ ft

$a = -k$

$v = -kt + C$

$88 = 0 + C$

$88 = C$

$v = -kt + 88$

when you've stopped at the end

$0 = -kt + 88$

$-88 = -kt$

$88 = kt$

$\frac{88}{k} = t$

$x = -\frac{k}{2}t^2 + 88t + C$

$0 = 0 + 0 + C$

$0 = C$

$x = -\frac{k}{2}t^2 + 88t$

$x = -\frac{k}{2}\left(\frac{88}{k}\right)^2 + 88\left(\frac{88}{k}\right)$

$242 = -\frac{88^2}{2k^2} + \frac{88^2}{k}$

$242 =$

$k = 16$

$a = -16$ ft/sec^2