

2.7/2.6/2.8

Differentiability implies continuity.  
The derivative exists iff  $\lim_{x \rightarrow a} f(x) = f(a)$ .

(Some) Rules of Differentiation

Constant:  $\frac{d}{dx} C = 0$

Constant Multiple:  $\frac{d}{dx}(Cu) = C \frac{d}{dx} u$

Sums & Diff:  $\frac{d}{dx}(u \pm v) = \frac{d}{dx} u \pm \frac{d}{dx} v$

Power:  $\frac{d}{dx}(x^n) = nx^{n-1}$

Product:  $\frac{d}{dx}(u \cdot v) = uv' + u'v$

Quotient:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - v'u}{v^2}$

Chain Rule:  $\frac{d}{dx}(f(u(x))) = f'(u) \cdot u'(x)$

Notation

$$y' = f'(x) = \frac{dy}{dx}$$

$$y'' = f''(x) = \frac{d^2y}{dx^2}$$

$$y''' = f'''(x) = \frac{d^3y}{dx^3}$$

$$y^{(iv)} = f^{(iv)}(x) = \frac{d^{(iv)}y}{dx^{(iv)}}$$

Ex #1 Find  $\frac{dy}{dx}$  for  $y = 4x^5 - 7x^2 + 2x$

$$\frac{dy}{dx} = 20x^4 - 14x + 2$$

Ex #2 Find the 2<sup>nd</sup>, 3<sup>rd</sup>, & 4<sup>th</sup> derivative from ex #1.

$$y'' = 80x^3 - 14$$

$$y''' = 240x^2$$

$$y^{(iv)} = 480x$$

Ex #3 Find  $y'$  for  $y = (x^2+1)(x^3+1)$

$$y' = 2x(x^3+1) + 3x^2(x^2+1) \leftarrow$$

$$y' = 2x^4 + 2x + 3x^4 + 3x^2$$

$$y' = 5x^4 + 3x^2 + 2x \leftarrow$$

Ex #4 Find the derivative of  $y = 2\sqrt{x} - \frac{1}{\sqrt{x}}$

$$y' = (x)^{-1/2} + \frac{1}{2}(x)^{-3/2}$$
$$y' = \frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{x^3}}$$

Ex #5 Find  $f'(x)$  for  $f(x) = \frac{3x^4 - 2x^3 + 4}{2x^2}$

$$f'(x) = \frac{(12x^3 - 6x^2)(2x^2) - (4x)(3x^4 - 2x^3 + 4)}{4x^4}$$
$$f'(x) = \frac{12x^5 - 4x^4 - 16x}{4x^4} = \frac{3x^4 - x^3 - 4}{x^3}$$

Ex #6 Find  $y'$  for  $y = (x^2 + x)^{1/2}$ .

$$y' = \frac{1}{2}(x^2 + x)^{-1/2}(2x + 1)$$