2.3 Definition of a Derivative

Derivative: At a point it represents the slope (instantaneous rate of change) of the tangent line.

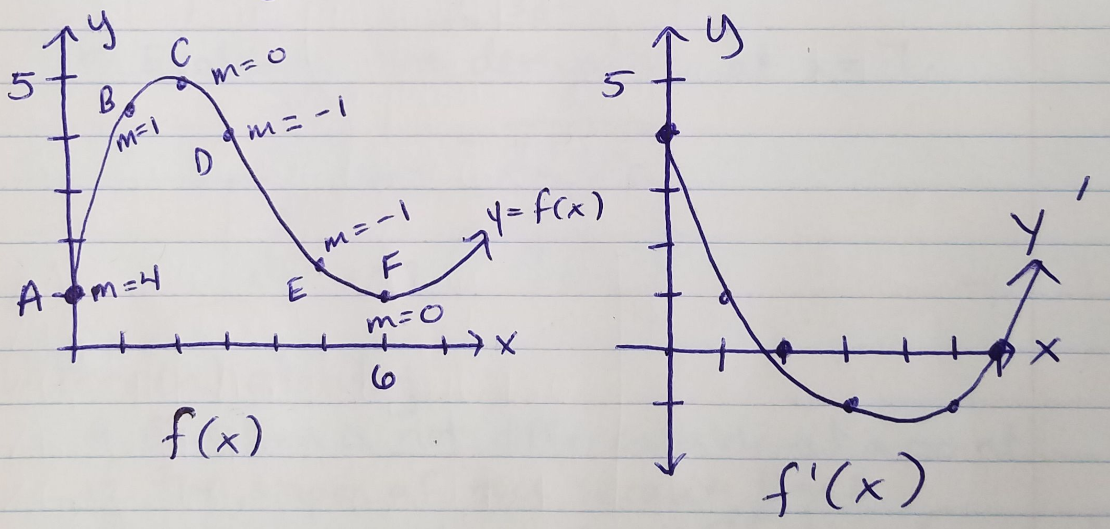
Notation

$y = f(x)$ the derivative is denoted by

Newton: $\begin{cases} f'(x) & \text{"f prime of x"} \\ y' & \text{"y prime"} \end{cases}$

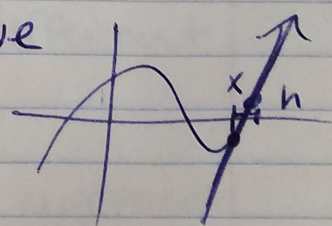
Leibniz: $\begin{cases} \dfrac{dy}{dx} & \text{"dy dx"} \\ \dfrac{df}{dx} & \text{"df dx"} \\ \dfrac{d}{dx} f(x) & \text{"d dx of f(x)"} \end{cases}$

Graphically



(Limit) Definition of Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



★ or alternatively ★

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

EX#1 $f(x)=\sqrt{3x-1}$

a) Eqn of the derivative

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3(x+h)-1}-\sqrt{3x-1}}{h}$$

mult. by conjugate

$$=\lim_{h\to 0}\frac{3(x+h)-1-(3x-1)}{h\left(\sqrt{3(x+h)-1}+\sqrt{3x-1}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{3x}+3\cancel{h}-\cancel{1}-\cancel{3x}+\cancel{1}}{\cancel{h}\left(\sqrt{3(x+h)-1}+\sqrt{3x-1}\right)}$$

$$=\lim_{h\to 0}\frac{3}{\sqrt{3(x+h)-1}+\sqrt{3x-1}}$$

$$\boxed{f'(x)=\frac{3}{2\sqrt{3x-1}}}$$

b) Evaluate the derivative at $x=7$.

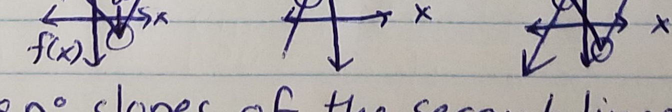
$$\left.\frac{df}{dx}\right|_{x=7}=\frac{3}{2\sqrt{20}}$$

$$\boxed{\left.\frac{df}{dx}\right|_{x=7}=\frac{3}{4\sqrt{5}}}$$

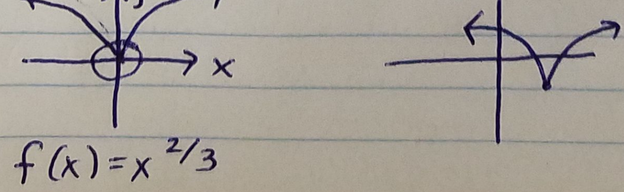
Differentiability

A function is not differentiable at a point if the slopes of the secant lines fail to approach a limit of f as $h\to 0$ (or $x\to a$).

corner: one-sided ~~limits~~ derivatives are different.

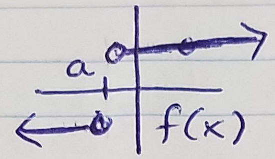


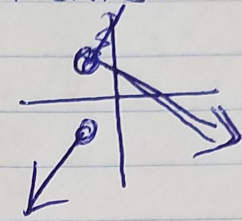
cusp: slopes of the secant lines approach $-\infty$ on one side & ∞ on the other.



$f(x)=x^{2/3}$

discontinuities: one or both sided derivatives do not exist





vertical tangent: slopes of secant lines <u>both</u> approach either ∞ or $-\infty$.