8.1 Conics: Parabolas

Vocabulary

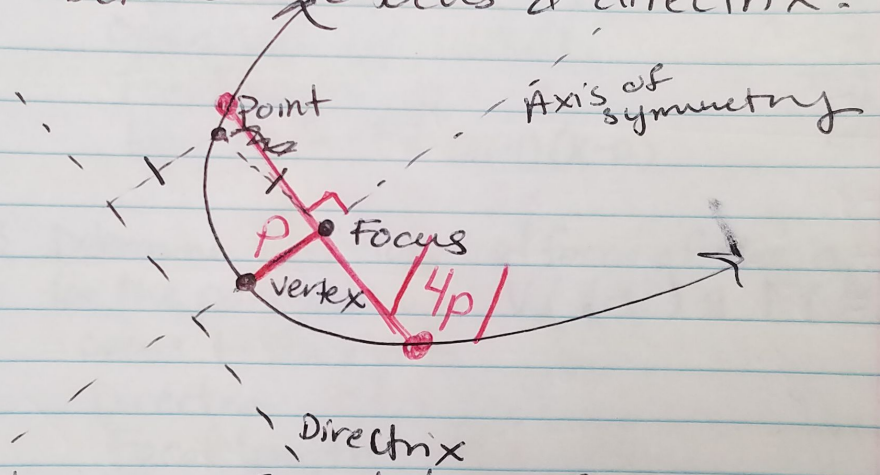
Conic section / Conic: A cross-section of a cone. Might have a parabola, an ellipse, or a hyperbola.

Parabola: The set of all points equidistant from a particular line (directrix) & a point (focus).

Directrix: A line on the "outside" of the parabola.

Focus: A point "inside" of the parabola.

Vertex: The point on the parabola that splits it into two equal sections. The vertex is between the focus & directrix.



Focal Length: The distance from the vertex to the focus. This is represented by p.

Focal width: Distance from one side of the parabola to the other that goes thru focus. $|4p|$

Parabolas w/ vertex (h,k)

Opens:	up/down	right/left
Equations:	$(x-h)^2=4p(y-k)$	$(y-k)^2=4p(x-h)$
Focus:	$(h, k+p)$	$(h+p, k)$
Directrix:	$y=k-p$	$x=h-p$
AOS:	$x=h$	$y=k$
Focal length:	p	p
Focal width:	$\|4p\|$	$\|4p\|$

Ex #1 Determine the focus, the directrix, & the focal width of $y = -\frac{1}{3}x^2$.

$x^2 = -3y$

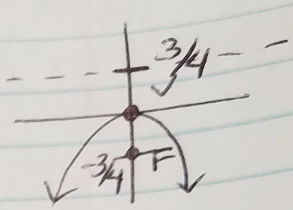
$V(0,0)$ opens down

$4p = -3 \rightarrow p = -3/4$

Focus: $(0, -3/4)$

Directrix: $y = 3/4$

Focal width: 3



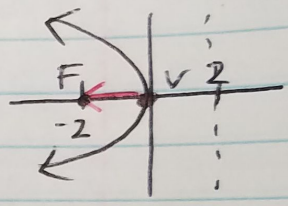
Ex #2 Determine the equation in std form of the parabola whose directrix is the line $x = 2$ and whose focus is $(-2,0)$.

Opens left w/ $V(0,0)$.

Focal length: $p = -2$

Equ: $(y-0)^2 = -2(4)(x-0)$

$y^2 = -8x$



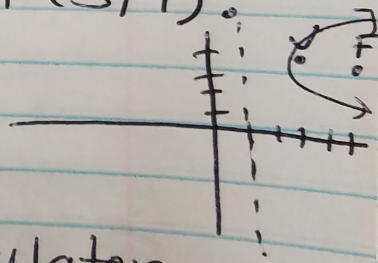
Ex #3 Determine the standard form of the equation for the parabola w/ $V(3,4)$ & $F(5,4)$.

Opens to the right

Directrix: $x = 1$

Focal length: $p = 2$

Equ: $(y-4)^2 = 8(x-3)$



Ex #4 Graph ex #3 on graphing calculator.

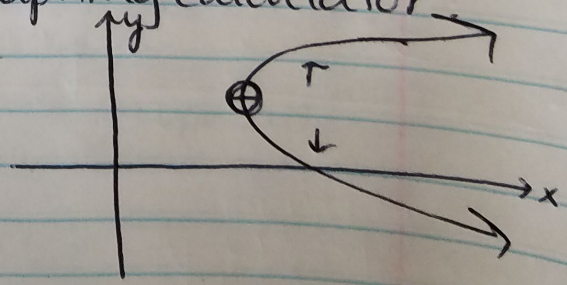
$(y-4)^2 = 8(x-3)$

$y - 4 = \pm\sqrt{8(x-3)}$

$y = 4 \pm \sqrt{8(x-3)}$

$y_1 = 4 + \sqrt{8(x-3)}$

$y_2 = 4 - \sqrt{8(x-3)}$



Ex#5 Prove that $y^2 - 6x + 2y + 13 = 0$ is a parabola. Find its vertex, focus, and directrix.

$y^2 - 6x + 2y + 13 = 0$

$y^2 + 2y + \boxed{1} = 6x - 13 + \boxed{1}$

$(y+1)^2 = 6x - 12$

$(y+1)^2 = 6(x-2)$

Vert: $(2, -1)$

Focus: $(3/2, -1)$

Directrix: $x = 1/2$

$p = \frac{3}{2} - 1$

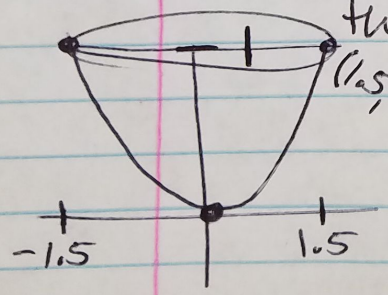
opens right

Applications

Main apps of parabolas: sound, light, radio waves, electro magnetic waves, etc.

Apps of paraboloids of revolution: flashlight, headlight, searchlights, microwave relays, television satellite dishes, reflecting telescopes, solar ovens, sideline microphones, etc.

Ex#6 A TV network uses a parabolic reflector with a microphone at the reflector's focus to capture conversations among players during a football game. If the reflector is 3 ft across & 1 ft deep, where should the microphone be placed?



Let a crosssection be an upward-facing parabola w/ V(0,0).

Eqn: $x^2 = 4py$

$(1.5)^2 = 4p(1)$

$p = \frac{1.5^2}{4}$

$p = \frac{2.25}{4}$

$p = 0.5625$ ft

$p = 6.75$ in

Place the microphone 6.75 in from the vertex along the axis of symmetry.