8.3 Conics: Hyperbolas

Vocabulary

Hyperbola: The set of all points whose distances from the foci have a ~~constant~~ constant difference.
Focal axis: The line through the foci.
Center: The point midway between the foci.
Vertices: The point(s) where the hyperbola intersects the focal axis.
Transverse axis: The line segment on the focal axis connecting two verts - length $2a$.
Conjugate axis: The line segment perpendicular to the focal axis that goes thru the center. length $2b$.

Hyperbolas w/ $C(h,k)$

orientation	horizontal	vertical
std eqn	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Focal axis	$y=k$	$x=h$
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Verts	$(h \pm a, k)$	$(h, k \pm a)$
semitransverse	a	a
semiconjugate	b	b
Pythagorean	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$
Asymptotes	$y = \pm \frac{b}{a}(x-h) + k$	$y = \pm \frac{a}{b}(x-h) + k$

EX#1 Find the verts & the foci of $4x^2 - 9y^2 = 36$.

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

horizontal hyperbola

$a = 3$

$b = 2$

$c = \sqrt{13}$

verts: $(\pm 3, 0)$

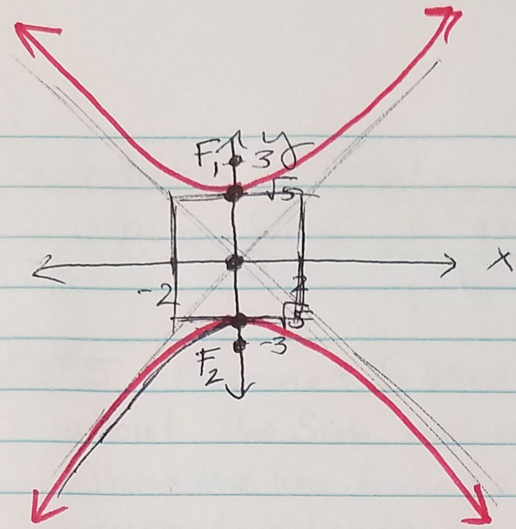
foci: $(\pm\sqrt{13}, 0)$

EX#2 Find the eqn of the hyperbola w/ foci $(0, \pm 3)$ whose conjugate axis has length 4. Sketch graph.

vert. hyperbola w/ $C(0,0)$

$a = \sqrt{5}$ $b = 2$ $c = 3$

$$\frac{y^2}{5} - \frac{x^2}{4} = 1$$



Ex# 3 Find the eqn of the hyperbola whose transverse axis has endpts $(-2,-1)$ & $(8,-1)$ & whose conjugate axis has length 8.

horizontal hyperbola

$C(3,-1)$

$a=5$

$b=4$

$$\frac{(x-3)^2}{25} - \frac{(\cancel{x}\,y+1)^2}{16} = 1$$

Ex#4 Find the center, verts, and foci of $\frac{(x+2)^2}{9} - \frac{(y-5)^2}{49} = 1$. sketch.

horizontal hyperbola

$a=3$

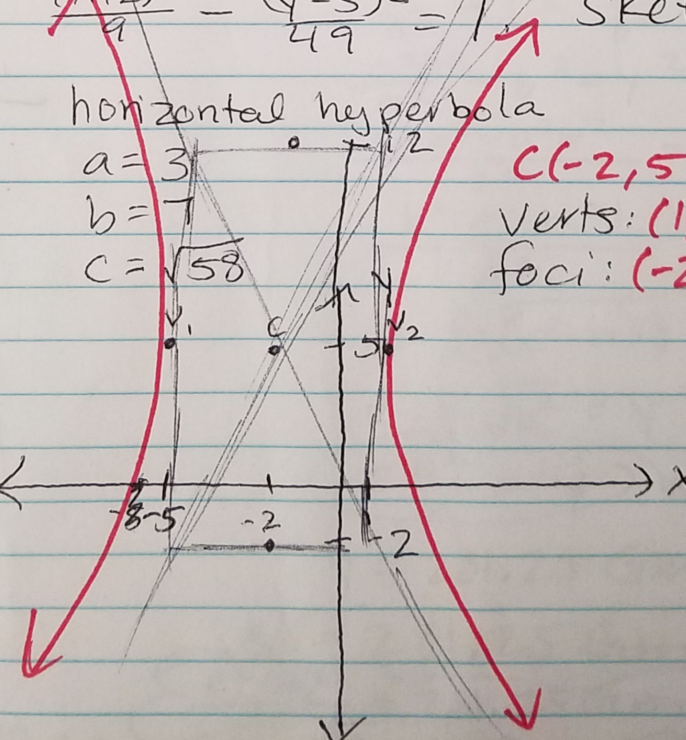
$b=7$

$c=\sqrt{58}$

$C(-2,5)$

verts: $(1,5)$ & $(-5,5)$

foci: $(-2\pm\sqrt{58},\ 5)$



Eccentricity & Orbits

$$e = \frac{c}{a} = \frac{\sqrt{a^2+b^2}}{a}$$

$e > 1$

Ex#5 A comet following a hyperbolic path about the Sun has a perihelion distance of 90 Gm. When the line from the comet to the Sun is perpendicular to the focal axis of the orbit $[x=c]$, the comet is 281.25 Gm from the Sun. Calculate a, b, c, & e & identify the coordinates of the center of the Sun.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

perihelion: $c - a = 90$ Gm

when $x = c$ $\rightarrow$ $\frac{c^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{c^2}{a^2} - 1 = \frac{y^2}{b^2}$$

$$\frac{c^2b^2}{a^2} - b^2 = y^2$$

$$\frac{c^2b^2 - a^2b^2}{a^2} = y^2$$

$$\frac{b^2(c^2-a^2)}{a^2} = y^2$$

$$\pm \frac{b}{a}\sqrt{(c^2-a^2)} = y$$

$$\pm \frac{b}{a}\sqrt{b^2} = y$$

$$\pm \frac{b^2}{a} = y$$ when $x=c$

$y = 281.25$ Gm

$$\frac{b^2}{a} = 281.25 \text{ Gm}$$

$$\frac{c^2-a^2}{a} = 281.25 \text{ Gm}$$

$c = 90 + a$

$$\frac{(90+a)^2-a^2}{a} = 281.25$$

$\vdots$

$a = 80$ Gm
$b = 150$ Gm $\quad (c^2=a^2+b^2)$
$c = 170$ Gm $\quad (c=90+a)$
$e = 17/8 \quad (e = c/a)$

Sun @ $(170, 0)$

Other Applications

Reflectors of sound, light, & other waves. Revolved about its focal axis, it is called a hyperboloid. A wave directed towards one focus & off one branch of the hyperbola will reflect in the direction of the other focus. The Hubble Space Telescope uses parabolic, hyperbolic, & elliptical mirrors to create ~~a~~ reflecting telescope.