8.2 Conics: Ellipses

Vocabulary

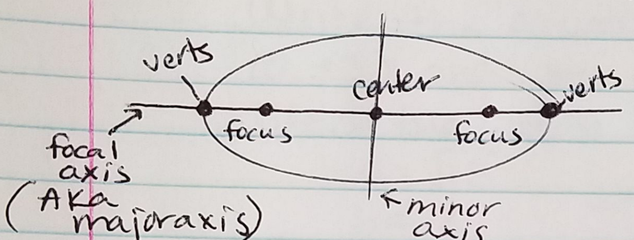
Ellipse: The set of all points whose distances from two fixed points (foci) have a constant sum.

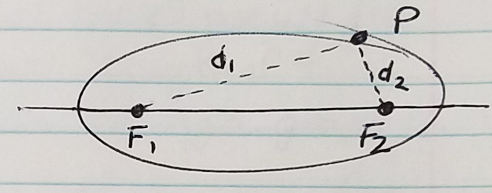
Foci: Plural of focus.

Focal axis: The line through the foci.

Center: The point midway of the focal axis.

Vertices: The points on the ellipse that intersect the focal axis.





$d_1 + d_2 = \text{constant}$

Major axis: AKA focal axis.

Minor axis: Perpendicular to the focal axis thru center.

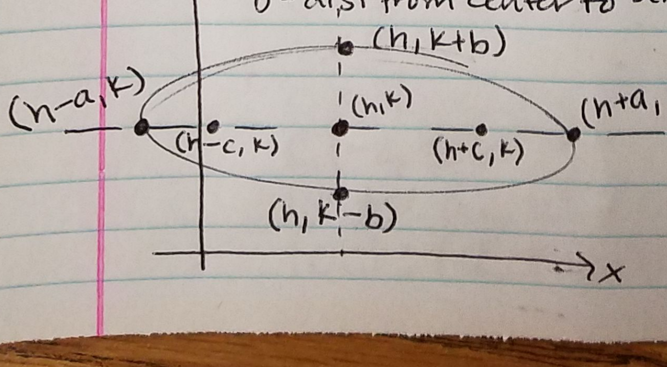
Ellipses w/ Center (h,k)

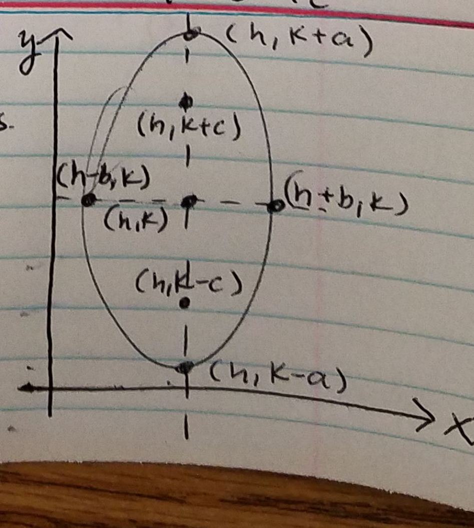
Type	Horizontal	Vertical
Eqn	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Focal axis	$y=k$	$x=h$
Foci	$(h\pm c, k)$	$(h, k\pm c)$
Verts	$(h\pm a, k)$	$(h, k\pm a)$
Semimajor axis	a (bigger #)	a (bigger #)
Semiminor axis	b (smaller #)	b (smaller #)
Pythagorean Relation	$a^2=b^2+c^2$	$a^2=b^2+c^2$

c = dist from center to focus

a = dist from center to vert.

b = dist from center to semi-verts.





Ex#1 Find the vertices and foci of $4x^2+9y^2=36$.

Std form: $\frac{x^2}{9}+\frac{y^2}{4}=1$

$a=3$ $\quad$ $a^2=b^2+c^2$

$b=2$ $\quad$ $9=4+c^2$

$c=\sqrt{5}$ $\quad$ $5=c^2$

verts: $(\pm 3,0)$

foci: $(\pm\sqrt{5},0)$

Ex#2 Write the equation of the ellipse w/ foci $(0,\pm 3)$ and a minor axis length of 4.
Sketch the graph & check using a grapher.

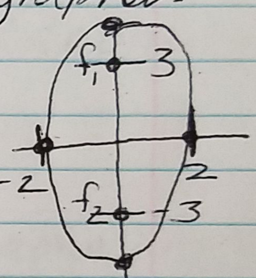
$a=\sqrt{13}$ $\quad$ $a^2=2^2+3^3$

$b=2$ $\quad$ $a^2=13$

$c=3$

$C(0,0)$

Eqn: $\frac{y^2}{13}+\frac{x^2}{4}=1$



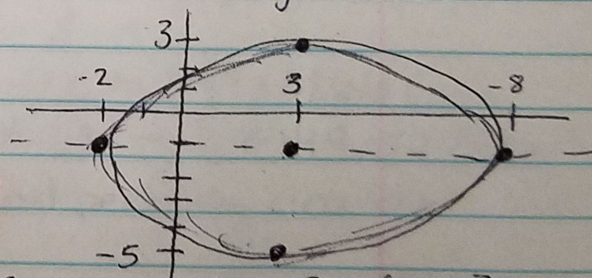
Ex#3 Find the equation in std form of the ellipse whose major axis has endpoints $(-2,-1)$ & $(8,-1)$ and whose minor axis has a length of 8.

$a=5$

$b=4$

$C(3,-1)$

$\frac{(x-3)^2}{25}+\frac{(y+1)^2}{16}=1$



Ex #4 Find the center, vets, and foci of $\frac{(x+2)^2}{9}+\frac{(y-5)^2}{49}=1$.
vertical ellipse

$a=7$

$b=3$

$c=\sqrt{40}=2\sqrt{10}$

$C(-2,5)$

verts: $(-2,12)$ & $(-2,-2)$

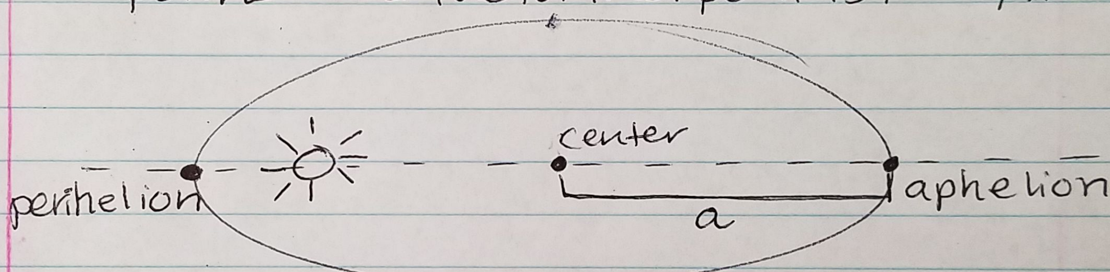
foci: $(-2,5\pm 2\sqrt{10})$

Orbits & Eccentricity

Eccentricity comes from the root word eccentric, which means off-center. It represents the ratio of c to a. Let $e = \frac{c}{a}$. If e is closer to zero, the ellipse is closer to being a circle. If e is closer to 1, the ellipse is very elongated.

$$e = \frac{c}{a} = \frac{\sqrt{a^2-b^2}}{a}$$

On an elliptical path around the sun, the closest point to the Sun is called the perihelion & the farthest point is the aphelion.



Ex #5 Earth's orbit has a semimajor axis of $a \approx 149.598$ Gm (gigameters) and an eccentricity of $e \approx 0.0167$. Calculate & interpret b & c.

$e = \frac{c}{a} \rightarrow c = ea \rightarrow c \approx 2.4982866$ Gm

$c^2 = a^2 - b^2 \rightarrow b = \sqrt{a^2 - c^2} \rightarrow b \approx 149.577$ Gm

The Earth has an almost perfectly circular elliptical orbit.

Other Applications

Reflectors of sound, light, & other waves. (Ellipsoid of revolution if revolved) Mirrors for optical equipment, study aircraft noise & wind tunnels.

Ex #6 A lithotripter breaks up kidney stones & is in the shape of an ellipsoid. One such device has a major axis = 12ft & a minor axis = 5ft. How far from are the foci?

$a = 6$ ft $b = 2.5$ ft

$c = \sqrt{6^2 - 2.5^2} \approx 5.4544$ ft ≈ 5 ft 5 in