

8.1/8.2 "i" and Operations with Complex Numbers

The Imaginary Number i

$$i = \sqrt{-1}$$

Ex #1 Write each number in terms of i.

a) $\sqrt{-25} = \sqrt{-1} \sqrt{25} = \boxed{5i}$

b) $\sqrt{-7} = \sqrt{-1} \sqrt{7} = \boxed{i\sqrt{7}}$

c) $\sqrt{-9} = \sqrt{-1} \sqrt{9} = \boxed{3i}$

d) $\sqrt{-12} = \sqrt{-1} \sqrt{3} \sqrt{4} = \boxed{2i\sqrt{3}}$

e) $\sqrt{-150} = \sqrt{-1} \sqrt{25} \sqrt{6} = \boxed{5i\sqrt{6}}$

f) $3 + \sqrt{-8} = 3 + \sqrt{-1} \sqrt{2} \sqrt{4} = \boxed{2i\sqrt{2} + 3}$

g) $5 - \sqrt{-36} = 5 - \sqrt{-1} \sqrt{36} = \boxed{5 - 6i}$

Complex Numbers

Can be real, imaginary, or both.

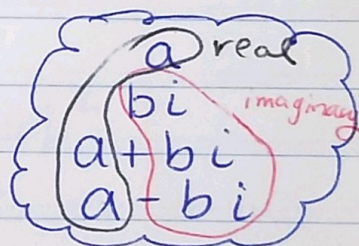
Ex #2 Identify ^{the} real and imaginary parts

a) $5 + 8i$ real: 5 imaginary: $8i$

b) 8 real: 8 imaginary: NONE

c) $i\sqrt{10}$ real: NONE imaginary: $i\sqrt{10}$

d) $\frac{5+3i}{2} = \frac{5}{2} + \frac{3i}{2}$ real: $\frac{5}{2}$ imaginary: $\frac{3i}{2}$



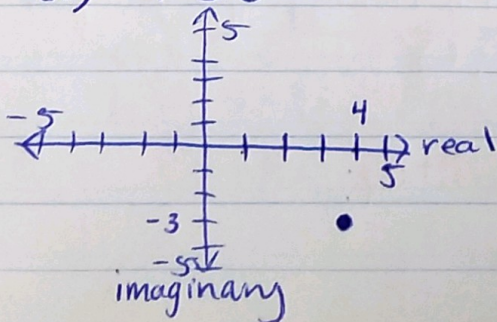
Complex Numbers: Graphing

x-axis → "real" axis

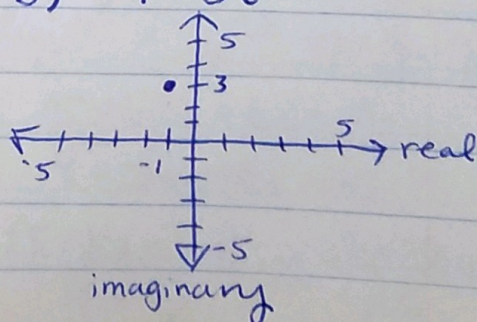
y-axis → "imaginary" axis

Ex #3 Graph the following numbers.

a) $4 - 3i$



b) $-1 + 3i$



Complex Numbers: Adding & Subtracting

Combine real w/ real & imaginary w/ imaginary.

Ex #4 Find the sum or difference & write in $a+bi$ form.

$$a) (6+5i) + (4-7i) = \boxed{10-2i}$$

$$b) (12-13i) - (-5+4i) = \boxed{17-17i}$$

$$c) (8-5i) - (3+5i) + (-5+10i) = \boxed{0+0i} = 0$$

Complex Numbers: Multiplying

Distribute/box & combine like terms. $i^2 = -1$

Ex #5 multiply and write in $a+bi$ form.

$$a) (6+5i)(4-7i) = 24-42i+20i-35i^2 \\ = 24-22i-35(-1) \\ = \boxed{59-22i}$$

$$b) (2-3i)(3-2i) = 6-4i-9i+6i^2 \\ = 6-13i-6 \\ = \boxed{-13i}$$

$$c) (8-4i)(5i) = 40i-20i^2 \\ = 40i+20 = \boxed{20+40i}$$

Complex Numbers: Dividing

Multiply top & bottom by the conjugate of the bottom.

conjugates: $a+bi$ & $a-bi$ (change middle \pm)

Ex #6 Divide & write in $a+bi$ form.

$$a) \left(\frac{7-3i}{i} \right) \left(\frac{-i}{-i} \right) = \frac{-7i+3i^2}{-i^2} = \frac{-7i-3}{1} = \boxed{-3-7i}$$

$$b) \left(\frac{5+2i}{3-4i} \right) \left(\frac{3+4i}{3+4i} \right) = \frac{15+20i+6i+8i^2}{9+12i-12i-16i^2} \\ = \frac{15+26i-8}{9+16} = \frac{7+26i}{25} = \boxed{\frac{7}{25} + \frac{26i}{25}}$$