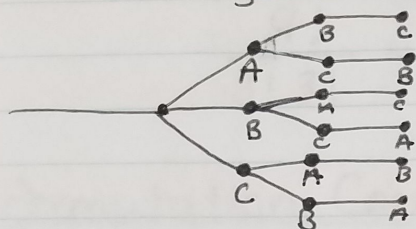## 9.1 Combinatorics

Ex #1: make a tree diagram to visualize the number of ways to arrange the letters A, B, & C.



A BC
AC B
B AC
BC A
CA B
CB A

6 ways to arrange

$$3 \times 2 \times 1 = 3! = 6$$

Ex #2: The CA license plates generally consist of 3 letters & 4 numbers. How many unique license plates are possible with these restrictions?

$$10 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10$$
$$= 10^4 \times 26^3$$
$$= 175,760,000 \text{ unique license plates}$$

Ex #3: What if no numbers or letters are repeated?

$$10 \times 26 \times 25 \times 24 \times 9 \times 8 \times 7$$
$$= 78,624,000 \text{ license plates}$$

## Permutations

The number of ways $n$ objects can be <u>arranged</u> <u>in order</u> (where abc is different than acb).

There are $n!$ permutations of a set of $n$ objects.

Ex #4: Count the number of ways you can arrange the word DRAGONFLY.

$$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 9! = 362,880 \text{ ways}$$

Ex #5: Count the number of ways you can arrange the word BUTTERFLY.

$$9!/2! = 181,440 \text{ ways}$$

Ex #6: Count the number of ways you can arrange
the word BUMBLEBEE.

$$\frac{9!}{3! \, 3!} = \frac{9 \times 8 \times 7 \times \cancel{6} \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$$

$$= 10,080 \text{ ways}$$

## Permutation Counting Formula

If you want the number of permutations of
n objects taken r at a time would be
denoted $_nP_r = P(n,r) = \frac{n!}{(n-r)!}$.

If r is greater than n, $_nP_r = 0$.

Ex #7: Sixteen actors answer a casting
call to try out for roles as dwarfs
in a production of Snow White.
In how many ways can the director
cast the seven roles?

$$_{16}P_7 = 57,657,600 \text{ ways}$$

## Combinations

The number of ways n objects can be selected
w/o regard to order would be denoted
$_nC_r = C(n,r) = \frac{n!}{r!(n-r)!}$.  If r is greater

than n, the number of combinations is zero.

Ex #8: Determine which of the following would be
permutations & which would be combinations.
a) A president, vice pres, secretary & treasurer
are chosen from a 25-member club. Permutation
b) You select 5 potatoes from a bag of 12 potatoes.
Combination
c) A teacher makes a seating chart for 22
students in a room w/ 30 desks.
Permutation

Ex #9: In the Miss America pageant, 51 contestants must be narrowed down to 10 finalists who will compete on nat'l TV. In how many ways can the 10 finalists be selected?

$$_{51}C_{10} = \frac{51!}{10!\,41!} = \frac{51\times50\times49\times48\times47\times46\times45\times44\times43\times42}{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}$$

$$= 12,777,711,870$$

Ex #10: Armando's Pizzeria offers patrons any combination of up to 10 different toppings. How many different pizzas can be ordered if we can choose

a) any three toppings?

$$_{10}C_3 = 120 \text{ ways}$$

b) any number of toppings?

$$_{10}C_0 + _{10}C_1 + _{10}C_2 + \cdots + _{10}C_{10}$$

<u>or</u>

$$\underset{\text{pepperoni}}{2} \times \underset{\text{mushroom}}{2} \times \underset{\text{sausage}}{2} \times \cdots - = 2^{10}$$

1024 ways to get any # of toppings

either you get it or you don't → 2 options