HW Questions

1.4 #33 In today's notes.

1.3 #22

1.4 Building Functions from Functions

Combinations of Functions

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(fg)(x) = f(x) \cdot g(x)$

$(f/g)(x) = f(x)/g(x) \rightarrow g(x) \neq 0$

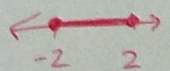
The domain of the new function of $f(x)$ is the intersection (overlap) between the domain of $f(x)$ and the domain of $g(x)$.

Ex #1 Let $f(x) = x^2$ & $g(x) = \sqrt{4-x^2}$

Find the combinations $f+g$, $f-g$, fg, & f/g and state their domain.

D_f: $(-\infty, \infty)$ D_g: $4-x^2 \geq 0$

$4 \geq x^2$



$[-2, 2]$

$(f+g)(x) = x^2 + \sqrt{4-x^2}$

$D: [-2, 2]$

$(f-g)(x) = x^2 - \sqrt{4-x^2}$

$D: [-2, 2]$

$(fg)(x) = x^2\sqrt{4-x^2}$

$D: [-2, 2]$

$f+g$

$\sqrt{4} \quad \sqrt{x^2}$

$\rightarrow \left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x^2}}$

$D: (-2, 2)$

Function Composition

$(f \circ g)(x) = f(g(x))$ "f of g of x"

Ex #2 Let $f(x) = 3x - 2$ & $g(x) = x - 1$

Find $(f \circ g)(x)$, $(g \circ f)(x)$ & their domains

& $(f \circ g)(2)$, $(g \circ f)(-3)$.

$D_f : (-\infty, \infty)$

$D_g : (-\infty, \infty)$

$(f \circ g)(x) = f(g(x))$

$= 3(x-1) - 2$

$= 3x - 3 - 2$

$\boxed{(f \circ g)(x) = 3x - 5}$ $D_{f \circ g} : (-\infty, \infty)$

$(f \circ g)(2) = 3(2) - 5$

$= 6 - 5$

$\boxed{(f \circ g)(2) = 1}$

$(g \circ f)(x) = g(f(x))$

$= (3x - 2) - 1$

$= 3x - 2 - 1$

$\boxed{(g \circ f)(x) = 3x - 3}$ $D_{g \circ f} : (-\infty, \infty)$

$(g \circ f)(-3) = 3(-3) - 3$

$= -9 - 3$

$\boxed{(g \circ f)(-3) = -12}$

Ex#3 Let $f(x)=x^2-2$ & $g(x)=\sqrt{x+1}$.

Find $(f\circ g)(x)$ & its domain.

$D_f: (-\infty,\infty)$ $D_g: [-1,\infty)$ $x+1\geq 0$, $x\geq -1$

$(f\circ g)(x)=f(g(x))$

$=\left(\sqrt{x+1}\right)^2-2$

$=x+1-2$

$(f\circ g)(x)=x-1$ $D_{f\circ g}: [-1,\infty)$

$(g\circ f)(x)=g(f(x))$

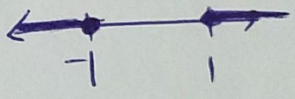
$=\sqrt{(x^2-2)+1}$

$(g\circ f)(x)=\sqrt{x^2-1}$

$D: (-\infty,-1]\cup[1,\infty)$

$x^2-1\geq 0$

$x^2\geq 1$



Domain of combinations you take the overlap.

Domain of compositions you must consider the domain of the input/inner function & the domain of the final composition.

Ex#4 Let $f(x)=\frac{x}{x+1}$ & $g(x)=4-x^2$

$D_f: (-\infty,-1)\cup(-1,\infty)$

$D_g: (-\infty,\infty)$

Do same as Ex#2

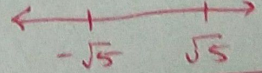
$(f\circ g)(x)=f(g(x))$

$=\frac{(4-x^2)}{(4-x^2)+1}$

$(f\circ g)(x)=\frac{4-x^2}{5-x^2}$

D: $5-x^2\neq 0$

$5\neq x^2$



$(f\circ g)(2)=0$

$=\frac{4-(2)^2}{5-(2)^2}=\frac{0}{1}$

$(-\infty,-\sqrt{5})\cup(\sqrt{5},\sqrt{5})\cup(\sqrt{5},\infty)$

$(-\infty,\infty)\rightarrow$ & $x\neq\sqrt{5},-\sqrt{5}$

$$(g\circ f)(x) = g(f(x))$$
$$= 4 - \left(\frac{x}{x+1}\right)^2$$
$$= 4 - \frac{x^2}{x^2+2x+1}$$
$$= \frac{4(x^2+2x+1) - x^2}{x^2+2x+1}$$
$$= \frac{4x^2+8x+4-x^2}{x^2+2x+1}$$

$$(g\circ f)(x) = \frac{3x^2+8x+4}{x^2+2x+1}$$

D: $(-\infty,-1)\cup(-1,\infty)$

$(g\circ f)(-3) = \frac{7}{4}$

Decomposition

Determine what functions, $f(x)$ & $g(x)$, created the composition given.

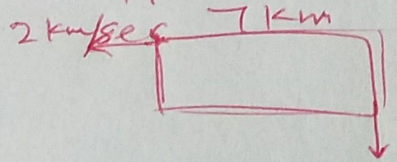
Ex #5 $y = \sqrt{x+1}$

$f(x) = \sqrt{x}$ $\quad g(x) = x+1$

Word Problem Ex

Ex #6 (#33 on your hw)

A satellite camera takes a rectangle-shaped picture. The smallest region that can be photographed is a 5-km by 7-km rectangle. As the camera zooms out, the length l and ~~width~~ w of the rectangle increase at a rate of 2 km/sec. How long does it take for the area A to be at least 5 times its original size?



$A_i = 5\text{ km} \times 7\text{ km} = 35\text{ km}^2$

$A_f = 5A_i = 5(35) = 175\text{ km}^2$ ← want

$A_f = (7+2t)(5+2t)$

$175 = (7+2t)(5+2t)$

$t \approx 3.633$ sec

$t \approx -9.633$ is an extraneous soln b/c we cannot do negative time

$$175 = 35 + 14t + 10t + 4t^2$$

$$0 = 4t^2 + 24t - 140$$

$$\frac{0 = t^2 + 6t - 35}{4}$$

Y_1 $\quad$ Y_2

-35, 6

Piecewise Functions

$$f(x) = \begin{cases} |x| & , \ x < 0 \\ 1 - x^2 & , \ x \geq 0 \end{cases}$$