

19.2 Arithmetic Series

Series - Adds up the terms in a sequence.

Sequences

$$3, 6, 9, 12, 15$$

$$4, 8, 16, 32, 64$$

$$-16, -14, -12, -10, -8$$

Series

$$3+6+9+12+15$$

$$4+8+16+32+64$$

$$(-16)+(-14)+(-12)+(-10)+(-8)$$

Partial Sum - Adds up the first n terms of a series, denoted S_n .

Ex#1 Consider 4, 6, 8, 10, 12, 14, 16, 18, ...

$$a) S_4 = 4+6+8+10 = 28$$

$$S_4 = 28$$

$$b) S_5 = 4+6+8+10+12 = 40$$

$$S_5 = 40$$

$$c) S_8 = 4+6+8+10+12+14+16+18 = 88$$

$$S_8 = 88$$

$$d) S_{10} = 4+6+8+10+12+14+16+18+20+22 = 130$$

$$S_{10} = 130$$

Finite Sum of an Arithmetic Series (Formulas)

↑ "Finite" means limited or that there is a beginning & an end, we will talk about some types of infinite sums later on.

Taking the sum of many terms (S_{100} or S_{1000}) is tedious, so you may use the following formulas:

$$S_n = \frac{n}{2} (a_1 + a_n) \quad \leftarrow \text{use this if you know the 1}^{\text{st}} \text{ \& } n^{\text{th}} \text{ terms of the series}$$

$$S_n = \frac{n}{2} (2a_1 + (n-1)d) \quad \leftarrow \text{use this if you know the 1}^{\text{st}} \text{ term and } d \text{ for the series}$$

Ex#2 Find S_{20} for the following:

$$a) 3+8+13+18+\dots$$

We know $a_1 = 3$ and $d = 5$, use 2nd formula

$$S_{20} = \frac{20}{2} (2(3) + (20-1)5)$$

$$= 10(6 + (19)5)$$

$$= 10(6 + 95)$$

$$= 10(101)$$

$$S_{20} = 1010$$



$$b) -2 - 4 - 6 - 8 - \dots$$

we know $a_1 = -2$ and $d = -2$ so use formula 2

$$\begin{aligned} S_{20} &= \frac{20}{2} (2(-2) + (20-1)(-2)) \\ &= 10 (-4 + (19)(-2)) \\ &= 10 (-4 - 38) \\ &= 10 (-42) \end{aligned}$$

$$\boxed{S_{20} = -420}$$

Ex #3 Find S_{49} for the following:

a) the arithmetic series with $a_1 = -4$ and $a_{49} = 110$

use the first formula since we know the 1st & 49th term

$$\begin{aligned} S_{49} &= \frac{49}{2} (-4 + 110) \\ &= \frac{49}{2} (106) \\ &= 49 \left(\frac{106}{2} \right) \\ &= 49 (53) \end{aligned}$$

★ you can also just do

$$\frac{49}{2} (106) = 24.5(106) = 2597$$

$$\boxed{S_{49} = 2597}$$

b) the arithmetic series with $a_1 = 10$ and $a_n = 3n + 7$

we know a_1 and a formula for a_n , use the formula to find a_{49}

$$a_{49} = 3(49) + 7 = 147 + 7 = 154$$

$$S_{49} = \frac{49}{2} (10 + 154) = 24.5(164) = 4018$$

$$\boxed{S_{49} = 4018}$$

A35: Arithmetic Series Worksheet