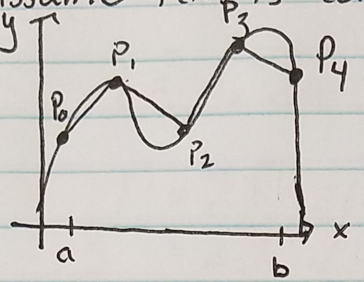
5.3/5.5

Arc Length

Assume $f(x)$ is continuous & differentiable on $[a,b]$.



Estimating the curve length by taking the sum of all the line segments represented by $\overline{P_{i-1} P_i}$

$$L = \lim_{n\to\infty} \sum_{i=1}^{n} |\overline{P_{i-1} P_i}|$$

$$|P_{i-1} P_i| = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$
$$= \sqrt{\Delta x^2 + \Delta y_i^2}$$

$\underline{y_i}$ varies!

★ MVT $\dfrac{f(b)-f(a)}{b-a} = f'(c)$

$$f(b)-f(a) = f'(c)(b-a)$$
$$f(x_i) - f(x_{i-1}) = f'(x^*)(x_i - x_{i-1})$$
$$\Delta y_i = f'(x^*)\Delta x$$

$$= \sqrt{\Delta x^2 + [f'(x^*)\Delta x]^2}$$
$$|P_{i-1} P_i| = \sqrt{1 + [f'(x^*)]^2}\,\Delta x$$

$$L = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + [f'(x)]^2}\,\Delta x$$

$$\boxed{\begin{gathered} L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx \\ \text{or} \\ L = \int_c^d \sqrt{1 + [h'(y)]^2}\,dy \end{gathered}}$$

Ex#1 (#1148) Find the length of the curve $x^2+y^2=1$ in two ways, one involving an integral.

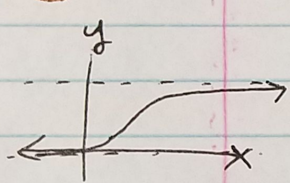
$$L=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

$$y=\pm\sqrt{1-x^2}$$
$$y'=\tfrac{1}{2}(1-x^2)^{-1/2}(-2x)$$
$$y'=\frac{-x}{\sqrt{1-x^2}}$$

$$L=2\int_{-1}^{1}\sqrt{1+\frac{x^2}{1-x^2}}\,dx$$
$$=2\int_{-1}^{1}\sqrt{\frac{1-\cancel{x^2}+\cancel{x^2}}{1-x^2}}\,dx$$
$$=2\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}\,dx$$
$$=2\arcsin x\Big|_{-1}^{1}$$
$$=2\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]$$
$$\boxed{L=2\pi}$$

$$S=r\theta$$
$$S=1(2\pi)$$
$$\boxed{S=2\pi}$$

Logistic Curve



$P(t)=\dfrac{M}{1+Ce^{-rMt}}$; M, C, r are costants

$\dfrac{dP}{dt}=r(M-P)P$; r & M are constants

(1) as $x\to\infty$, $P(t)\Rightarrow M$

(2) $\frac{dP}{dt}$ changes fastest when $P=\frac{M}{2}$