7.5

Tests that will always work (alternaiting or otherwise)

#1	n^{th} Term Test	$\lim_{n\to\infty} a_n \neq 0$ then diverges
#2	Ratio Test	$L = \lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert$
#2.5	Root Test	$L = \lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert}$

$L>1$ diverges
$L<1$ converges absolutely
$L=1$ " sucks

#3 try other tests w/ abs. values

Absolute Convergence

Σa_n converges absolutely if $\Sigma\lvert a_n\rvert$ converges.

Conditional Convergence

Σa_n converges conditionally if $\Sigma\lvert a_n\rvert$ diverges and Σa_n converges.

$\Sigma a_n = \Sigma(-1)^{n+1} b_n$

Alternaiting Series Test

If b_n is positive, decreasing, and $\lim_{n\to\infty} b_n = 0$ then $\Sigma(-1)^{n+1} b_n$ converges.

Ex#1 $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

$$\lim_{n\to\infty}\frac{(0.1)^n}{n} = 0$$

$$L = \lim_{n\to\infty}\left\lvert\frac{(0.1)^{n+1}}{n+1}\left(\frac{n}{(0.1)^n}\right)\right\rvert = \lim_{n\to\infty}\left\lvert\frac{0.1n}{n+1}\right\rvert = 0.1$$

$L<1$

$\boxed{\text{Series converges absolutely by Ratio test.}}$

Ex#2 $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$

$$\lim_{n\to\infty}\frac{n}{n^3+1} = 0$$

$$L = \lim_{n\to\infty}\left\lvert\frac{n+1}{(n+1)^3+1}\left(\frac{n^3+1}{n}\right)\right\rvert = \lim_{n\to\infty}\left\lvert\frac{n^4+n^3+n+1}{n^4+3n^3+3n^2+2n}\right\rvert = 1$$

must determine if $\sum_{n=1}^{\infty}\frac{n}{n^3+1}$ converges

$\sum_{n=1}^{\infty}\frac{n}{n^3+1} < \sum_{n=1}^{\infty}\frac{1}{n^2}$ converges by p-series

$\boxed{\text{Series converges absolutely by DCT.}}$

Ex#3 $\sum_{n=1}^{\infty}(-1)^n\frac{\sin n}{n^2}$

$$-\sum_{n=1}^{\infty}\frac{1}{n^2} \le \sum_{n=1}^{\infty}\frac{\sin n}{n^2} \le \sum_{n=1}^{\infty}\frac{1}{n^2}$$

converges by p-series — converges by p-series

$\boxed{\text{Series converges absolutely by p-series \& Sandwich theorem/DCT.}}$

Ex#4 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$

$\lim_{n\to\infty} \frac{1+n}{n^2} = 0$ ✓

$\sum_{n=1}^{\infty} \frac{1+n}{n^2} > \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by direct comparison to harmonic

$\sum_{n=1}^{\infty} \frac{1+n}{n^2} = \frac{2}{1} + \frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \frac{6}{25} + \cdots$

terms are positive ✓ terms are decreasing ✓

a_n converges by AST.

$\boxed{\text{Series converges conditionally}}$

Ex#5 $\sum_{n=1}^{\infty} (-1)^{n+1} \sqrt[n]{10}$

$\lim_{n\to\infty} \sqrt[n]{10} = \lim_{n\to\infty} 10^{\frac{1}{n}} = 1 \neq 0$

$\boxed{\text{Series diverges by } n^{th} \text{ term Test.}}$

Estimation Error

Can calculate sums: Geometric $\frac{a}{1-r}$ $|r|<1$

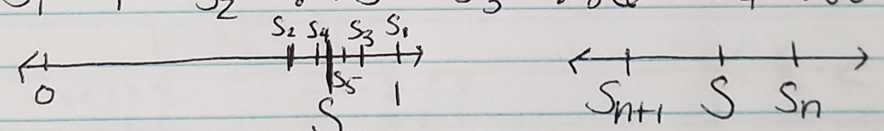
Telescoping terms cancel out

Alternating Sums $\longrightarrow$ Σa_n satisfies AST,

then $\Sigma a_n = S$

mini ex: $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^2}\right) = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \cdots$

$S_1 = 1$ $S_2 = .75$ $S_3 = .86$ $S_4 = .80$ $S_5 = .84$

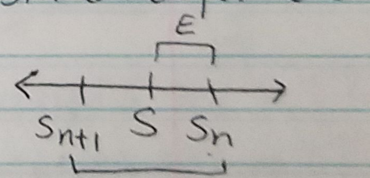


E = Estimation error = |actual sum − estimated partial sum|

$E = |S - S_n|$

$E \leq |S_{n+1} - S_n|$

$\boxed{E \leq |a_{n+1}|}$



Ex#6 Estimate the error in using the first 15 terms to approximate the sum of $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$.

$E \leq |a_{n+1}|$

$E \leq \left|(-1)^{n+2} \frac{1}{(n+1)^2}\right|$

$E \leq \left|(-1)^{17} \frac{1}{16^2}\right|$

$E \leq 0.0039\ldots$

$\boxed{E \leq 0.004}$