

A59: + & - Rational & Complex Fractions

29.2 Add & Subtract Rational Expressions & Complex Fractions

- ① Factor & identify restrictions (when denom = 0)
- ② Get common denominators by multiplying numerator & denominator by any "missing" factors.
- ★ Keep the denominator factored★
- ③ Just like with regular fractions, combine numerators but leave the denominator alone/the same.

<p>Ex #1 $\frac{3}{x+1} - \frac{x}{x-1}$ $x \neq \pm 1$</p> $= \frac{3(x-1)}{(x+1)(x-1)} - \frac{x(x+1)}{(x-1)(x+1)}$ $= \frac{3x-3}{(x+1)(x-1)} - \frac{x^2+x}{(x+1)(x-1)}$ $= \frac{3x-3-x^2-x}{(x+1)(x-1)}$ <div style="border: 1px solid red; padding: 5px; display: inline-block;"> $= \frac{-x^2+2x-3}{(x+1)(x-1)} ; x \neq \pm 1$ </div>	<p>Ex #2 $\frac{2}{x} - \frac{3}{x^2-3x}$ $x \neq 0, 3$</p> $= \frac{2(x-3)}{x(x-3)} - \frac{3}{x(x-3)}$ $= \frac{2x-6}{x(x-3)} - \frac{3}{x(x-3)}$ $= \frac{2x-6-3}{x(x-3)}$ <div style="border: 1px solid red; padding: 5px; display: inline-block;"> $= \frac{2x-9}{x(x-3)} ; x \neq 0, 3$ </div>
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<p>Ex #3 $\frac{2}{x^2-4} + \frac{x}{x^2+4x+4}$ $x \neq \pm 2$</p> <del style="color: red; font-size: small;">$\frac{-2}{0} \frac{-4}{2}$ $= \frac{2(x+2)}{(x-2)(x+2)(x+2)} + \frac{x(x-2)}{(x+2)(x+2)(x-2)}$ $= \frac{2x+4}{(x-2)(x+2)(x+2)} + \frac{x^2-2x}{(x-2)(x+2)(x+2)}$ <div style="border: 1px solid red; padding: 5px; display: inline-block;"> $= \frac{x^2+4}{(x-2)(x+2)^2} ; x \neq \pm 2$ </div>	<p>Ex #4 $\frac{x}{x+2} + \frac{4}{x-3}$ $x \neq -2, 3$</p> $= \frac{x(x-3)}{(x+2)(x-3)} + \frac{4(x+2)}{(x-3)(x+2)}$ $= \frac{x^2-3x}{(x+2)(x-3)} + \frac{4x+8}{(x+2)(x-3)}$ <div style="border: 1px solid red; padding: 5px; display: inline-block;"> $= \frac{x^2+x+8}{(x+2)(x-3)} ; x \neq -2, 3$ </div>
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Complex Fractions

- ① Get one fraction in the numerator & one fraction in the denominator of the "bigger" fraction
- ② Factor & identify restrictions on anything except the numerator of the numerator
- ③ Multiply by the reciprocal of the denominator (of big fraction) and simplify

Ex#5 $1 + \frac{1}{x+1}$

$$x - \frac{x}{x-1}$$

$$= \frac{1(x+1)}{1(x+1)} + \frac{1}{x+1}$$

$$= \frac{x(x+1) - x}{1(x-1)}$$

$$= \frac{x+1}{x+1} + \frac{1}{x+1}$$

$$= \frac{x^2 - x - x}{x-1}$$

$$= \frac{x+2}{x+1} = \frac{x+2}{x+1} \cdot \frac{x-1}{x(x-2)}$$

$$= \frac{x+2}{x+1} \left(\frac{x-1}{x(x-2)} \right)$$

$$= \frac{(x+2)(x-1)}{x(x+1)(x-2)} ; x \neq \pm 1, 0, 2$$

Ex#6 $\frac{x}{x+1} - \frac{1}{x-1}$

$$\frac{1}{x+1} + 2$$

$$= \frac{x(x-1)}{(x+1)(x-1)} - \frac{1(x+1)}{(x-1)(x+1)}$$

$$= \frac{1}{x+1} + \frac{2(x+1)}{1(x+1)}$$

$$= \frac{x^2 - x - x + 1}{(x+1)(x-1)}$$

$$= \frac{1}{x+1} + \frac{2x+2}{x+1}$$

$$= \frac{x^2 - 2x - 1}{(x+1)(x-1)}$$

$$= \frac{2x+3}{x+1}$$

$$= \frac{x^2 - 2x - 1}{(x+1)(x-1)} \left(\frac{x+1}{2x+3} \right)$$

$$= \frac{x^2 - 2x - 1}{(x-1)(2x+3)} ; x \neq \pm 1, -\frac{3}{2}$$