Notes: 31.1/31.2 Arc Length and Radians

Radian – An angle measurement, similar to degrees (like how inches are similar to centimeters, or how miles are similar to kilometers).

Radian to Degree: $\left(\frac{180°}{\pi}\right)$ Degree to Radian: use $\left(\frac{\pi}{180°}\right)$

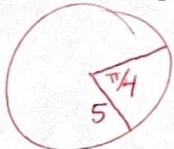1. Convert the following to radians.

   a. 30°

   $30°\left(\frac{\pi}{180°}\right) = \frac{1}{6}\pi = \boxed{\frac{\pi}{6} \text{ rad}}$

   b. 45°

   $45°\left(\frac{\pi}{180°}\right) = \frac{1}{4}\pi = \boxed{\frac{\pi}{4} \text{ rad}}$

   c. 360°

   $360°\left(\frac{\pi}{180°}\right) = \boxed{2\pi \text{ rad}}$

2. Convert the following to degrees.

   a. $\frac{\pi}{3}$

   $\frac{\pi}{3}\left(\frac{180°}{\pi}\right) = \frac{180°}{3} = \boxed{60°}$

   b. $\frac{5\pi}{6}$

   $\frac{5\pi}{6}\left(\frac{180°}{\pi}\right) = \frac{5(180°)}{6} = 5(30°) = \boxed{150°}$

   c. $\frac{3\pi}{2}$

   $\frac{3\pi}{2}\left(\frac{180°}{\pi}\right) = \frac{3(180°)}{2} = 3(90°) = \boxed{270°}$

Arc Length – The length of a section of the circumference of a circle. The length of the arc depends on the central angle and radius of the circle.

θ in degrees: $S = \frac{\pi r\theta}{180°}$ θ in radians: $S = r\theta$

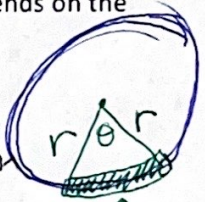3. What is the length of an arc from a circle with a central angle of $\frac{\pi}{4}$ and a radius of 5 inches?
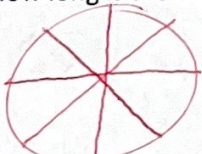
   $S = r\theta$ "in terms of π"   approximation

   $S = 5(\pi/4)$

   $S = \frac{5\pi}{4} \text{ inches} = 1.25\pi \text{ inches} = 3.927 \text{ inches}$ arc

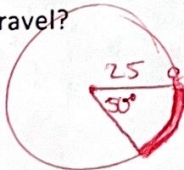4. How long is the crust on a pizza slice on a 16 inch diameter and the whole pizza has 8 slices?

   $r = 8 \text{ inches}$

   $\theta = 45° \left(= \frac{360°}{8}\right)$

   $S = \frac{\pi(8)(45°)}{180°}$
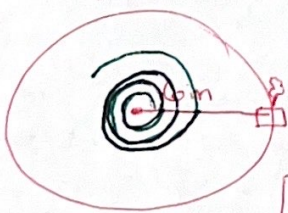
   $\boxed{S = 2\pi \text{ inches} = 6.283 \text{ inches}}$

5. A rider on a Ferris wheel is 25 feet from the center and the wheel rotates 50°. How much did the rider travel?

   $S = \frac{\pi(25)(50°)}{180°}$

   $\boxed{S = 6.944\pi \text{ feet} = 21.817 \text{ feet}}$

6. A rotating desert case does a full rotation every 9 minutes. How far will a desert item travel in 30 minutes if placed 6 inches from the center of the case?

   $9 + 9 + 9 + 3 = 30 \text{ min}$

   $360° + 360° + 360° + 120° = 1200°$

   $S = \frac{\pi(6)(1200°)}{180°}$

   $\boxed{S = 40\pi \text{ inches or } 125.664 \text{ inches}}$