6.1/6.2 Inverses

Inverses: By Definition (Composition)

The functions $f(x)$ and $g(x)$ are inverses iff

$$f(g(x)) = x \quad \underline{\text{AND}} \quad g(f(x)) = x.$$

Ex #1 Are $f(x) = 30 + 80x$ and $g(x) = \dfrac{x-30}{80}$ inverses?

$$f(g(x)) = 30 + 80\left(\frac{x-30}{80}\right) \qquad g(f(x)) = \frac{(30+80x)-30}{80}$$

$$= 30 + x - 30 \qquad\qquad = \frac{80x}{80}$$

$$= x \ \smile \qquad\qquad\qquad = x \quad \smile \boxed{\text{Yes}}$$

Ex #2 Are $f(x) = -3x + 8$ and $g(x) = 3x - 8$ inverses?

$$f(g(x)) = -3(3x - 8) + 8$$

$$= -9x + 24 + 8 \qquad \boxed{\text{Not inverses}}$$

$$= -9x + 32 \quad \overset{\parallel}{\frown}$$

Ex #3 Are $f(x) = \frac{1}{4}(x+12)$ and $g(x) = 4x - 12$ inverses?

$$f(g(x)) = \frac{1}{4}\left((4x - 12) + 12\right) \qquad g(f(x)) = 4\left(\frac{1}{4}(x+12)\right) - 12$$

$$= \frac{1}{4}(4x) \qquad\qquad\qquad = (x + 12) - 12$$

$$= x \ \smile \qquad\qquad\qquad = x \ \smile \ \boxed{\text{Yes}}$$

Inverses: Algebraically

Switch x's and y's, then solve for y.

Ex #4 Find the inverse of $f(x) = -3x + 8$.

$$y = -3x + 8$$

$$x = -3y + 8$$

$$x - 8 = -3y$$

$$\frac{x-8}{-3} = y$$

$$\frac{8-x}{3} = y$$

$$\boxed{f^{-1}(x) = \frac{8-x}{3}}$$

EX#5 Find the inverse of $h(x) = \frac{2}{3}x - 5$.

$$y = \frac{2}{3}x - 5$$

$$x = \frac{2}{3}y - 5$$

$$x + 5 = \frac{2}{3}y \qquad \text{ALL are correct}$$

$$3(x+5) = 2y$$

$$\frac{3}{2}(x+5) = y$$

$$\boxed{h^{-1}(x) = \frac{3}{2}(x+5)} = \frac{3x+15}{2} = \frac{3(x+5)}{2} = \frac{3}{2}x + 7.5$$

EX#6 Find the inverse of $g(x) = \frac{3x-2}{6}$.

$$y = \frac{3x-2}{6}$$

$$x = \frac{3y-2}{6}$$

$$6x = 3y - 2$$

$$6x + 2 = 3y \qquad \text{also correct}$$

$$\frac{6x+2}{3} = y$$

$$\boxed{g^{-1}(x) = \frac{6x+2}{3}} = 2x + \frac{2}{3}$$

Ex#7 Find the inverse of $j(x) = 5(x-1)$.

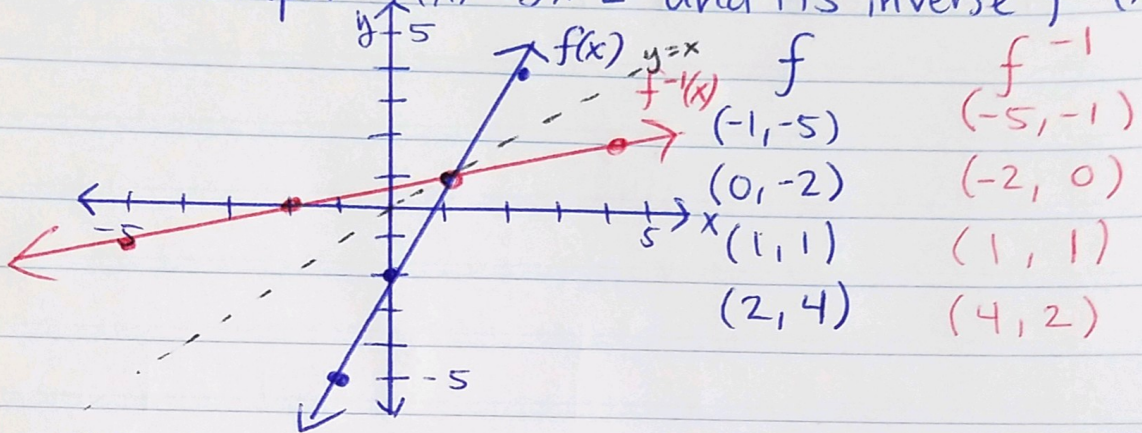$$y = 5(x-1)$$

$$y = 5x - 5$$

$$x = 5y - 5$$

$$x + 5 = 5y$$

$$\frac{x+5}{5} = y$$

$$\boxed{j^{-1}(x) = \frac{x+5}{5}} = \frac{x}{5} + 1$$

Inverses: Graphically

If (x, y) is a point on f, then (y, x) is a point on f^{-1}. The graphs of f and f^{-1} are reflections over the line $y = x$.

Ex #8 Graph $f(x) = 3x - 2$ and its inverse $f^{-1}(x)$.

f	f^{-1}
(-1, -5)	(-5, -1)
(0, -2)	(-2, 0)
(1, 1)	(1, 1)
(2, 4)	(4, 2)

Ex #9 Graph $f(x) = 6 - 3x$ and its inverse $f^{-1}(x)$.

f	f^{-1}
(0, 6)	(6, 0)
(1, 3)	(3, 1)
(2, 0)	(0, 2)
(3, -3)	(-3, 3)
(4, -6)	(-6, 4)