Probability and Statistics

7

Unit Overview

In this unit you will investigate whether a normal distribution is an appropriate model for data and, if it is, how to use the model to analyze and understand the data. You will learn the importance of impartiality in surveys and experiments, as well as use simulations to decide whether data are consistent or inconsistent with a conjecture. You will also investigate how to use data from a randomized experiment to compare two treatments and decide if an observed treatment effect is statistically significant.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- placebo
- simulation

Math Terms

- density curve
- z-score
- normal distribution
- normal curve
- sample
- survey
- response
- bias
- simple random sample
- experiment
- explanatory variable
- response variable
- completely randomized design
- randomized block design
- matched pairs design
- single-blind study
- double-blind study
- observational study
- confounding variable
- simulation
- statistic
- margin of error
- sample proportion
- sampling distribution
- critical value
- statistically significant

ESSENTIAL QUESTIONS

- What role does a random process play when conducting a survey?
- What role does a random process play when conducting an experiment with two treatments?
- How can a simulation help you decide if a set of data is consistent or inconsistent with a conjecture about the world?

EMBEDDED ASSESSMENTS

This unit has two embedded assessments, following Activities 37 and 40. These assessments will allow you to demonstrate your understanding of the relationships between data and models of real-world situations.

Getting Ready

Write your answers on notebook paper.
Show your work.

1. The following are the lengths of time, in minutes, that it took each member of a group of 12 running buddies to complete a marathon.

241	229	230	234	215	231
239	229	221	231	220	238

   a. Make a stem-and-leaf plot of the data, using ten-minute intervals for the stems.
   b. Make a dot plot of the data.
   c. Make a histogram of the data using five-minute intervals.
   d. Describe the distribution of the data using everyday language.
   e. Use technology to determine the mean and median of the 12 marathon times.
   f. Suppose these 12 friends were joined by a thirteenth running buddy who completed the marathon in 205 minutes. Describe how that runner compares to the other twelve.

2. Suppose that 12 families with one child each were surveyed and asked these questions: "About how much time, in minutes, do you spend reading to your children each week?" and "How tall is your child, in inches?" If a strong negative correlation were observed in a scatter plot of (reading time, height), would that imply that reading to your children stunts their growth? Explain.

Normal Distribution

ACTIVITY 36

Take Me Out to the Ballgame

Lesson 36-1 Shapes of Distributions

Learning Targets:

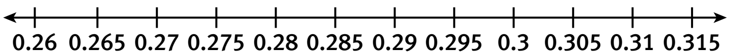
- Represent distribution with appropriate data plots.
- Interpret shape of a distribution and relate shape to measures of center and spread.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Activating Prior Knowledge, Interactive Word Wall, Create Representations, Look for a Pattern, Think-Pair-Share, Group Presentation, Jigsaw, Quickwrite, Self Revision/Peer Revision

The sport of baseball has a long history of players, fans, and management maintaining and interpreting players' statistics. One of the most common statistics used to describe a hitter's effectiveness is the *batting average*. Batting average is defined as the number of hits a player achieves divided by the number of at-bats that the player needs to achieve those hits. Work with your group on Items 1–4.

1. A local recreational baseball club, the Cobras, has twelve players. The batting averages for those players are as follows. 0.265, 0.270, 0.275, 0.280, 0.280, 0.280, 0.285, 0.285, 0.285, 0.285, 0.290, 0.290.

a. Create a dot plot for the batting averages.



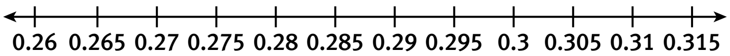
b. Describe the shape of the dot plot.

c. Find the mean and median of the data set. Which is larger?

d. What is the connection between the shape of the distribution and the location of the mean and median in the distribution?

2. Another local baseball club, the Manatees, also has 12 players. The batting averages for those players are as follows. 0.275, 0.275, 0.280, 0.280, 0.280, 0.280, 0.285, 0.285, 0.285, 0.290, 0.295, 0.305.

a. Create a dot plot for the batting averages.



b. Describe the shape of the dot plot.

c. Find the mean and median of the data set. Which is larger?

d. What is the connection between the shape of the distribution and the location of the mean and median in the distribution?

My Notes

CONNECT TO SPORTS

$$\text{Batting Average} = \frac{\text{Number of Hits}}{\text{Number of At-Bats}}$$

For example, if a player gets four hits in ten at-bats, then the batting average is $\frac{4}{10} = 0.400$. (Batting averages are reported rounded to the nearest thousandth.)

DISCUSSION GROUP TIP

Reread the problem scenario as needed. Make notes on the information provided in the problem. Respond to questions about the meaning of key information. Summarize or organize the information needed to create reasonable solutions, and describe the mathematical concepts your group will use to create its solutions.

MATH TIP

When a graphical representation shows that data has a "tail" in one direction, the data is described as skewed in the direction of the tail (either left or right). With skewed data, the mean is "pulled" away from the median in the direction of the skew. The mean will be close to the median if the data is not skewed and has no outliers.

My Notes

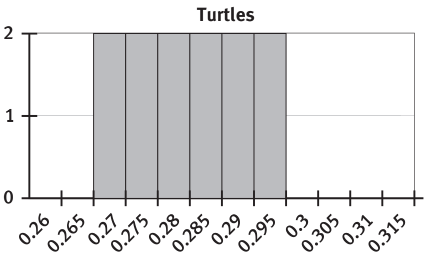
3. Compare and contrast the shapes of the distributions of batting averages for the Cobras and the Manatees. How are the characteristics of the distributions related to the measures of center, the mean, and the median?

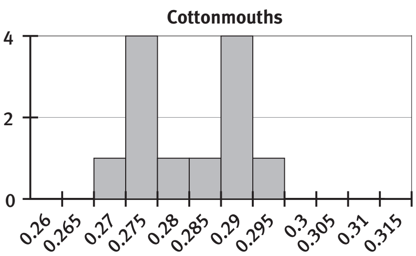
4. Find the standard deviation of the batting averages for the Cobras and the standard deviation of the batting averages for the Manatees. What do these standard deviations measure?

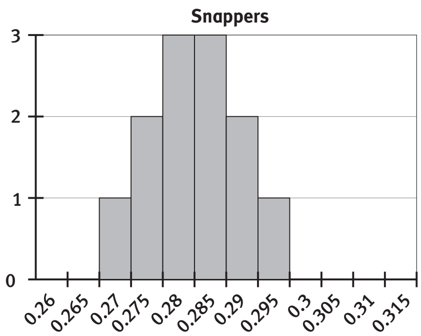
MATH TIP

Use technology to determine the standard deviation. On a TI graphing calculator, input the data in a list, and then press $\boxed{\text{STAT}}$, go to CALC, and select *1:1-Var Stats* to calculate the standard deviation (use the *Sx* = value). Alternatively, you can use the formula on page 635.

Three other teams, the Turtles, the Cottonmouths, and the Snappers, have their batting average data displayed in the histograms below.







5. Compare and contrast the histograms of these three teams.

MATH TIP

Data can be described as unimodal if it has one maximum in a graphical representation. This is true even if the data has two numerical modes as seen here for the Snappers. Data with two local maxima can be described as bimodal.

6. Find the mean and median for each of these distributions. How are your results related to the distributions?

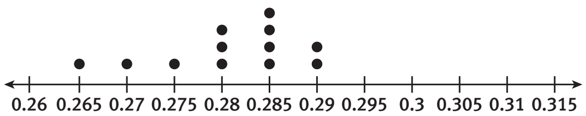
My Notes

7. Guess which team's distribution of batting averages has the largest standard deviation. Guess which one has the smallest. Use your calculator to find the actual standard deviations and confirm, or revise, your conjectures.

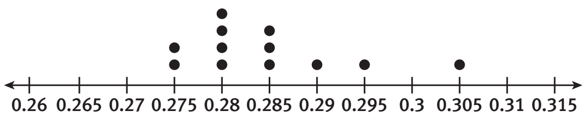
If a distribution follows a well-defined pattern, a smooth curve can be drawn to represent the distribution.

8. Below are each of the distributions that we saw in Items 1–7. For each distribution, draw a smooth curve on the distribution that best represents the pattern.

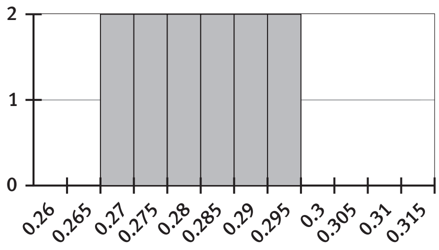
Cobras



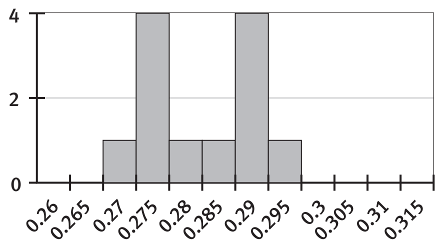
Manatees



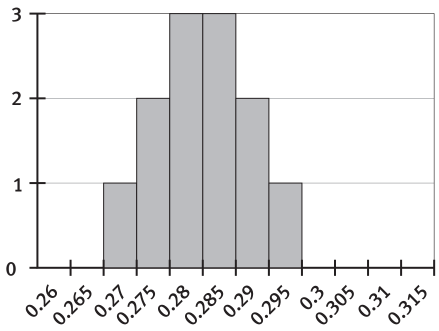
Turtles



Cottonmouths



Snappers



Each of the curves drawn above is called a ***density curve***. Density curves have special characteristics:

- Density curves are always drawn above the x-axis.
- The area between the density curve and the x-axis is always 1.

My Notes

On the Cobras, the player with the batting average of 0.270 was Walter. One player with a batting average of 0.290 was Leslie. The coach wanted to know how each player compared to the mean batting average.

From your previous work, you discovered that the mean batting average for the Cobras was 0.2808 and that the standard deviation was 0.0076. The coach performed the following calculations:

$$\frac{0.270 - 0.2808}{0.0076} = -1.421; \frac{0.290 - 0.2808}{0.0076} = 1.211$$

9. Work with your group on this item and on Item 10. Describe the meaning of each number in the calculations. What do the results of the calculations represent?

DISCUSSION GROUP TIP

As you share your ideas for Items 9 and 10, be sure to use mathematical terms and academic vocabulary precisely. Make notes to help you remember the meaning of new words and how they are used to describe mathematical concepts.

10. What is the meaning of the positive or negative sign in the result of the calculations?

The numbers that are the results of the coach's calculations are called ***z-scores***. Such scores standardize data of different types so that comparisons to a mean can be made.

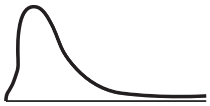
MATH TIP

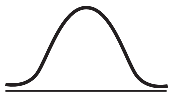
$$z\text{-score} = \frac{x - \text{mean}}{\text{standard deviation}}$$

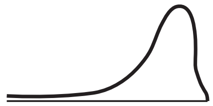
My Notes

Check Your Understanding

11. The grades on a quiz for three of Mr. Dean's classes were analyzed by finding the mean, standard deviation, and shape of the distribution for each class. Mr. Dean dropped his papers after doing this analysis, and the shapes of the distributions were separated from the means and medians. Which shape belongs to which mean and median?

A.


B.


C.


a. Mean: 70
Median: 70

b. Mean: 70
Median: 60

c. Mean:70
Median: 80

12. The mean length of a python is 2 m with a standard deviation of 0.3 m. The mean weight of the same species of python is 25 kg with a standard deviation of 5.4 kg. A 2.7 m python weighing 30 kg is captured in a state park. Use *z*-scores to determine which characteristic of the snake is more unusual: its length or its width. Explain your reasoning.

LESSON 36-1 PRACTICE

Charles has a jar in which he places any pennies that he may obtain during his daily activities. His sister, Oluoma, takes a handful of pennies off the top and records the dates on the pennies:
2013, 2012, 2008, 2012, 2011, 2013, 2012, 2013, 2011, 2011, 2010, 2009, 2012, 2013, 2012, 2010.

13. What is the mean date of the pennies? What is the median of the dates?

14. Draw a dot plot of the data, and then draw a smooth density curve that represents the data. Describe the shape of the distribution and its relation to your responses in Item 13.

15. Find the standard deviation of the penny date data, and determine the *z*-score for the dates of 2012 and 2009. What is the significance of the sign of the *z*-score of each?

Lesson 36-2
Characteristics of the Normal Distribution

My Notes

Learning Targets:
- Recognize characteristics of a normal distribution.
- Use mean and standard deviation to completely describe a normal distribution.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Summarizing, Close Reading, Marking the Text, Activating Prior Knowledge, Interactive Word Wall, Create Representations, Look for a Pattern, Think-Pair-Share, Group Presentations, Jigsaw, Quickwrite, Self Revision/Peer Revision, Create a Plan, Debrief

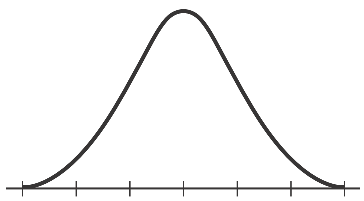
Consider the distribution of batting averages for the Snappers baseball team. Recall that the distribution is symmetrical, unimodal, and somewhat bell-shaped. Distributions with such characteristics are frequently considered to be ***normal distributions***. The density curves for these distributions are called ***normal curves***. Normal curves are special, as the mean and standard deviation provide a complete description of the distribution.

The distribution of team batting averages for the St. Louis Cardinals for the 50 years from 1964 to 2013 can be considered approximately normal. The mean batting average for these years is 0.2637, and the standard deviation is 0.0096.

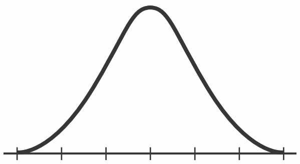
1. What is the median batting average for the St. Louis Cardinals for the years 1964–2013? Explain your reasoning.

To determine a scale when drawing a normal curve, it is important to note that the mean value corresponds to the peak of the curve and that the points at which the curve changes from *concave up* to *concave down* (or vice versa) are approximately one standard deviation from the mean. (These points are called *inflection points*.)

2. Use the mean and standard deviation for the St. Louis Cardinals batting average data from 1964–2013 to label the three middle tic marks on the scale for the normal curve below. Explain how you chose to label the scale.



MATH TIP

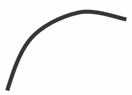


The normal curve

MATH TIP

Concave up looks like:

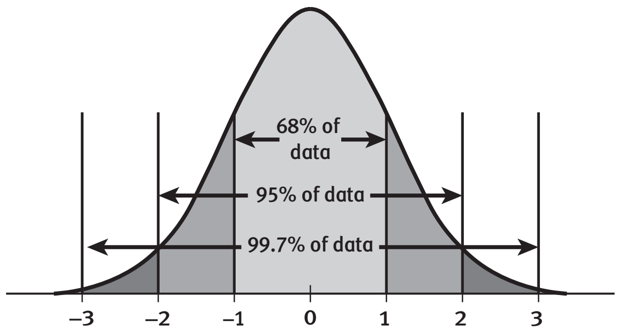
Concave down looks like:



CONNECT TO AP

The terms *concave up*, *concave down*, and *inflection points* are important in the study of Calculus.

As mentioned previously, normal distributions are completely described by the mean and standard deviation. The *68-95-99.7 rule* further reinforces this fact. This rule states that, in a normal distribution, approximately 68% of the data lies within one standard deviation of the mean, 95% of the data lies within two standard deviations of the mean, and 99.7% of the data lies within three standard deviations of the mean. This powerful fact is illustrated in the diagram below.



My Notes

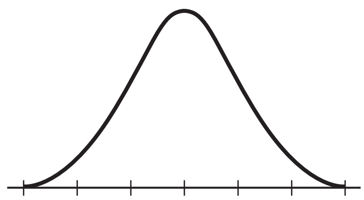
MATH TIP

In statistics, when talking about a *percent* of a data set, it is customary to use the word *proportion*.

For example: The proportion of data that lies within one standard deviation of the mean is 0.68.

3. Consider your normal curve from Item 2. What percent (proportion) of the data lies between the two data points that were not identified as the mean? Write a sentence about the team batting average of the St. Louis Cardinals that uses the 68-95-99.7 rule and these two data points.

4. Complete the scale for the normal curve in Item 2.



5. Between what two batting averages are 95% of the data? 99.7% of the data? For this 50-year period, in how many years would you expect the team batting average to be outside three standard deviations?

My Notes

MATH TIP

How to read the Standard Normal Table: For a given *z*-score, look in the left-hand column to find the row with the appropriate units and tenths digit. On the top row, find the column with the appropriate hundredths digit. Find the cell that is in both the row and column you identified. The four-digit decimal number in this cell represents the proportion of the normal distribution below the *z*-score.

TECHNOLOGY TIP

How to use the normalcdf function on a TI-84 graphing calculator: Press 2nd VARS for the distribution menu, and then press 2 for *normalcdf*. On your home screen, *normalcdf* (will appear. Enter "*−100, z-score, 0, 1*)" so that the command looks like *normalcdf* (*−100, z-score, 0,1*), and press ENTER. This will yield the proportion of the normal distribution below the *z*-score.

MATH TIP

Recall that the area under a density curve is one. Therefore, all numbers on the Standard Normal Table represent areas that are equivalent to the proportion less than a specific *z*-score.

6. Consider the question, "What proportion of the St. Louis team batting averages for the years 1964–2013 is below 0.269?"

a. Why is 67%, the average of 50% (the mean) and 84%(one standard deviation above), an incorrect response?

b. What difficulty exists in answering this question?

Recall that a *z*-score is the number of standard deviations above or below the mean. In a normal distribution, the *z*-score becomes extremely valuable thanks to the *Standard Normal Table*, or *z*-table. This table is found at the end of this activity, and it provides the area under the normal curve up to a specified *z*-score. Your graphing calculator can also provide you with results from the Standard Normal Table.

7. Use the Standard Normal Table to answer the following items.

a. Find the *z*-score for the batting average of 0.269. Round your *z*-score to the nearest hundredth.

b. Locate the *z*-score on the Standard Normal Table, and write the area that corresponds to the *z*-score.

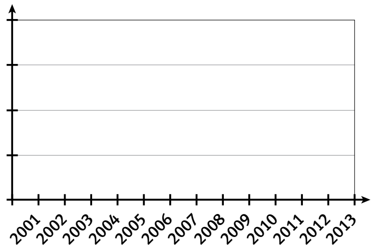
8. Use the rounded *z*-score you found in 7a and your graphing calculator to find the area and compare it to your result in 7b. Write the calculator syntax of the instruction and the answer, rounded to four decimal places.

Lesson 36-2
Characteristics of the Normal Distribution

My Notes

Check Your Understanding

Charles and Oluoma took all 900 pennies out of their penny jar and gathered information on their dates. They created a histogram of their data that is displayed below.



9. Oluoma claimed that the distribution was approximately normal. On what evidence did she base her claim?

10. Charles figured that the mean of the penny date data is 2007, and the standard deviation is 2.5.

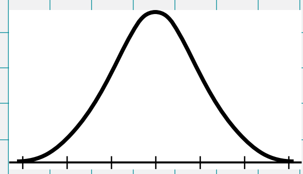
a. What is the median of the data?

b. How many pennies lie within one standard deviation of the 2007?

c. The *z*-score for a penny dated 2005 is −0.7561. Without computing, find and interpret the *z*-score for a 2009 penny.

LESSON 36-2 PRACTICE

11. A rock and sand supplier packages all-purpose sand in 60-pound bags. A sample of 200 bags was analyzed, and the distribution of actual weights was approximately normal, with a mean of 61 pounds and a standard deviation of 0.75 pounds. Use the 68-95-99.7 rule to complete the scale on the normal curve shown.



12. Evaluate the *z*-score for a 61.75-pound bag of all-purpose sand, and find the corresponding proportion in the *z*-table. Does this agree with the 68-95-99.7 rule? Explain your reasoning.

13. Evaluate the *z*-score for a bag of sand weighing 59.5 pounds. Using the *z*-table, find the proportion that corresponds to that *z*-score. What does this proportion imply?

14. With the same *z*-score from Item 13, use your graphing calculator to find the proportion for the 59.5-pound bag. Does this agree with your answer from Item 13?

15. Consider a 62-pound bag of all-purpose sand from this sample.

a. Evaluate the *z*-score for this bag of sand. Using the 68-95-99.7 rule, between which two proportions must this *z*-score correspond?

b. Use your *z*-score and the *z*-table to find the proportion that corresponds to the *z*-score.

c. Use your *z*-score and your calculator to find the proportion that corresponds to the *z*-score.

d. Use the proportions you found in Items 15b and 15c to describe the proportion of bags that weigh less than 62 pounds and the proportion that weighs more than 62 pounds.

My Notes

Learning Targets:

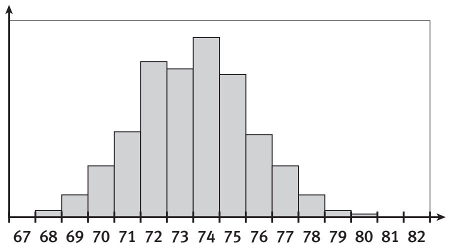
- Estimate probabilities associated with z-scores using normal curve sketches.
- Determine probabilities for z-scores using a standard normal table.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Summarizing, Close Reading, Marking the Text, Activating Prior Knowledge, Interactive Word Wall, Create Representations, Look for a Pattern, Think-Pair-Share, Group Presentation, Identify a Subtask

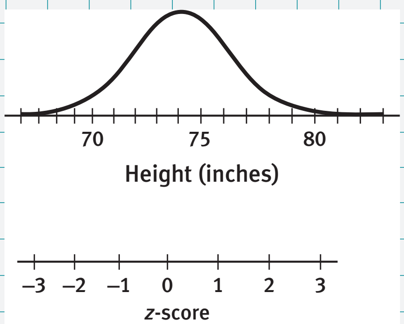
WRITING MATH

The lowercase Greek letter μ (pronounced "myew") is commonly used to represent the mean of a population. The lowercase Greek letter σ (pronounced "sigma") is commonly used to represent the standard deviation of a population.

The histogram below displays the heights, rounded to the nearest inch, of all Major League Baseball players in the year 2012.



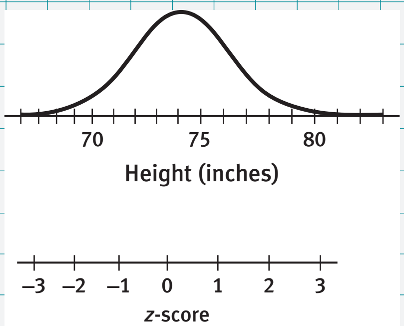
The shape of the graph—symmetric, unimodal, and bell-shaped—indicates that it would be reasonable to model the heights with a normal distribution. The mean and standard deviation of these players' heights are, respectively, $\mu = 73.5$ inches and $\sigma = 2.25$ inches. A picture of a normal density curve having this mean and standard deviation is shown. There are two scales given for the distribution. The upper scale is in inches, and the lower scale is in z-scores. (Remember that a z-score measures the number of standard deviations from a data point above or below the mean.)



1. One baseball player, Kevin Mattison, is 6'0" (72 inches) tall.
   a. Compute and interpret the z-score corresponding to his height.

   b. On the graph shown, draw a vertical line at Kevin Mattison's height, and shade the region under the bell curve and to the left of the vertical line you drew.

   c. The area of the region you shaded, when compared to the area of the entire region underneath the normal curve, corresponds to those players who are as tall as, or shorter than, 72 inches. Just by looking at the picture, estimate what proportion of players satisfies this condition.

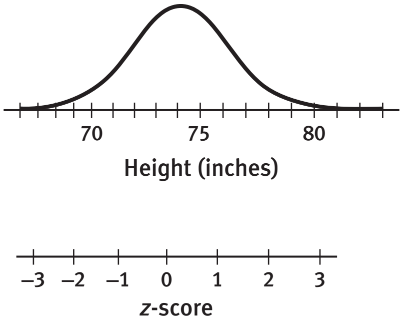


2. Another baseball player, Jose Ceda, is 6'4" (76 inches) tall.
   a. What is the z-score corresponding to his height?

My Notes

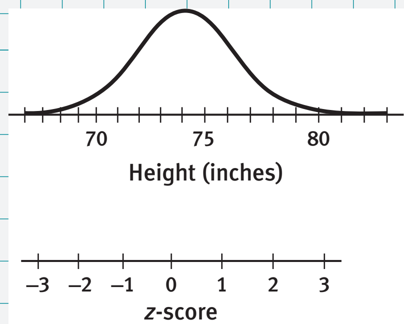
b. Interpret the meaning of the *z*-score you found in part a.

c. On the graph below, draw a vertical line at Jose Ceda's height, and shade lightly the region under the bell curve and to the right of your vertical line.



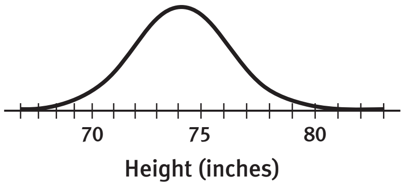
d. The region you shaded, when compared to the entire region underneath the bell curve, corresponds to those players who are as tall as, or taller than, 76 inches. Just by looking at the picture, estimate what proportion of players satisfies this condition.

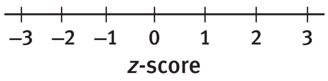
3. Suppose you are interested in the proportion of players' heights that, when rounded to the nearest inch, will be 6'3" (75 inches). Those are the players whose heights range from 74.5 inches to 75.5 inches. Compute the *z*-scores for both endpoints of that range. Then draw vertical lines at those locations on the graph, shade the region between the lines, and estimate the proportion of players' heights to which the area of the region corresponds.



My Notes

4. One baseball player, Dan Jennings, is taller than 80% of all other players. Draw a vertical line in the graph below at his height, and shade the region that corresponds to the proportion of players who are shorter than Dan. Then estimate Dan Jennings's height and the corresponding z-score.





There are four different kinds of estimates you made above, all relative to a distribution of values that is approximately normal:

- Estimating the proportion of the distribution that is less than a given value,
- Estimating the proportion of the distribution that is greater than a given value,
- Estimating the proportion of the distribution that lies between two given values,
- Estimating the value that has a given proportion of the population below it.

There are other variations, but if you master the skills associated with finding good estimates in these four situations, you should be able to handle other similar situations.

My Notes

You have already seen one way to estimate these values: sketching a normal curve and guessing, just by looking, what proportion of the total area beneath the curve lies in certain regions. Two other methods for making these estimates are more exact: using a Standard Normal Table (z-table) and technology. Even when using these two methods, it is always appropriate to sketch a normal curve and shade the region of interest.

Using the Standard Normal Table (z-Table)

A z-table shows the proportion of a standard normal probability distribution that is less than a particular z-score for many possible values of z. Recall that the area under the normal curve is one, so the values in the z-table also refer to the area under the normal curve to the left of a z-score as well. Use the z-table at the end of this activity.

5. Work with your group on this item and on Items 6–8. In Item 1, you computed the z-score corresponding to 72-inch-tall Kevin Mattison. Look in the z-table for the z-score that you computed and find the proportion of the distribution that is less than that z-score. Your answer should be similar to the value that you guessed in Item 1.

DISCUSSION GROUP TIP

With your group, reread the problem scenarios as needed. Make notes on the information provided in the problems. Respond to questions about the meaning of key information. Organize the information needed to create reasonable solutions, and describe the mathematical concepts your group uses to create its solutions.

6. In Item 2, you computed the z-score corresponding to 76-inch-tall Jose Ceda. Look in the z-table for the z-score you computed and find the proportion of the distribution that is less than that z-score. Then use that proportion to address the question, "What proportion of players is *taller* than Jose Ceda?"

My Notes

7. In Item 3, you estimated the proportion of players whose heights were between 74.5 and 75.5 inches, and so would round their heights to 75 inches. Use the z-table to estimate that proportion.

8. In Item 4, you estimated the height of Dan Jennings, given that he is taller than 80% of Major League Baseball players. Use the z-table to estimate his z-score, and use this z-score to estimate his height.

Check Your Understanding

9. If you estimated the proportion of baseball players' heights that, when rounded to the nearest inch, are 80 inches, would you expect that fraction to be larger, smaller, or about the same as the fraction of players whose heights, rounded to the nearest inch, are 73 inches? Explain your answer without doing any computations.

10. In the z-table, if a probability (area) is less than 0.50, what must be true about its corresponding z-score? Why?

11. When using the z-table, sometimes you look up a z-score in the table and then find the corresponding number in the body of the table. At other times, you look up a number in the body of the table and find the corresponding z-score. How do you know which of these is the right thing to do?

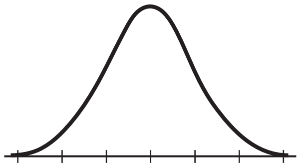
My Notes

LESSON 36-3 PRACTICE

All members of the junior class at a local high school took the PSAT exam. The distribution of the results of the mathematics section was found to be approximately normal, with a mean score of 52 and a standard deviation of 6.8.

12. Andres got a 55 on the mathematics section of the exam.

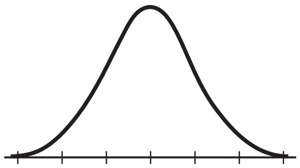
a. On the normal curve below, shade the proportion of students that scored less than or equal to Andres's score.



b. Evaluate the z-score for Andres's score and use the z-table to write the proportion of students that received a score less than or equal to 55.

13. Amber got a 60 on the mathematics section of the exam.

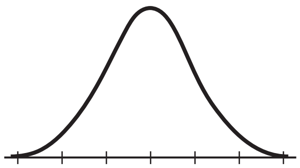
a. On the normal curve below, shade the proportion of students that scored greater than or equal to Amber's score.



b. Evaluate the z-score for Amber's score and use the z-table to write the proportion of students that received a score greater than or equal to 60.

14. Ms. Diaz, the assistant principal, made a quick review of the scores and commented that, based on her observation, it seemed most students scored between 50 and 56.

a. On the normal curve below, shade the proportion of students that scored between 50 and 56.



b. Evaluate the z-scores for PSAT math scores of 50 and 56, and then use the z-table to write the proportion of students that received scores between 50 and 56.

c. Confirm or revise Ms. Diaz's comment regarding the scores of the PSAT math section.

15. Stephan claimed that he scored better than 90% of the students in the junior class. Use z-scores and your z-table to determine what score Stephan must have earned to be correct.

My Notes

Learning Targets:

- Determine probabilities for z-scores using technology.
- Use a normal distribution, when appropriate, as a model for a population from which a sample of numeric data has been drawn.

SUGGESTED LEARNING STRATEGIES: Summarizing, Marking the Text, Activating Prior Knowledge, Create Representations, Look for a Pattern, Think-Pair-Share, Group Presentation, Jigsaw, Quickwrite, Self Revision/Peer Revision, Create a Plan, Identify a Subtask

Many calculators and computer spreadsheets can compute proportions of normal distributions directly, without first having to compute a z-score. (Keep in mind that the z-score still has a meaning and is useful in its own right.) Here you will see how to perform those computations using the TI-84.

To find the fraction of a normal distribution lying between any two values, we use this command:

$$\text{normalcdf}(L, U, \mu, \sigma),$$

where:

- L is the lower (lesser) of the two values,
- U is the upper (greater) of the two values,
- μ is the mean of the normal distribution, and
- σ is the standard deviation of the normal distribution.

Example A

To find the fraction of Major League Baseball players who would round their heights to 75 inches, you would enter:

$$\textit{normalcdf}(74.5, 75.5, 73.5, 2.25)$$

Answer: 0.1413

Try These A

a. Evaluate *normalcdf*(73.5, 76.5, 73.5, 2.25) on your calculator and interpret what each value represents in terms of the Major League Baseball player context.

b. Use your calculator to find the proportion of Major League Baseball players that are between 70 inches and 73 inches tall.

Lesson 36-4
Modeling with the Normal Distribution

My Notes

If you are interested in an interval of heights that has no lower bound, use the same command but with a very low number for *L*, the lower bound, a number that is well below any reasonable value in the distribution.

Example B

To find the proportion of players who are shorter than Kevin Mattison, use the following syntax. Notice that *L* is 0 in this example. In the context of heights of baseball players, such a value is unreasonably small, making it an appropriate lower bound.

normalcdf(0, 72, 73.5, 2.25)

Answer: 0.2525

Try These B

a. Evaluate *normalcdf*(−100, 76, 73.5, 2.25) on your calculator and interpret what each value represents in terms of Major League Baseball player heights.

b. Use your calculator to find the proportion of these players that are shorter than 72 inches.

If you are interested in an interval of heights that has no upper bound, use the same command but with a very high number for *U*, the upper bound, a number that is well above any reasonable value in the distribution.

Example C

To find the proportion of players who are taller than Jose Ceda, use the following syntax. Notice that *U* is 1000 in this example. In the context of heights of baseball players, such a value is unreasonably large, making it an appropriate upper bound.

normalcdf(76, 1000, 73.5, 2.25)

Answer: 0.1334

Try These C

a. Evaluate *normalcdf*(75, 200, 73.5, 2.25) on your calculator and interpret what each value represents in terms of Major League Baseball player heights.

b. Use your calculator to find the proportion of these players who are taller than 70 inches.

My Notes

For situations in which you know the proportion of the distribution below an unknown value, the command used to find the unknown value is

invNorm(p, μ, σ) where:

- p is the fraction of the distribution that is less than the desired value,
- μ is the mean of the normal distribution, and
- σ is the standard deviation of the normal distribution.

TECHNOLOGY TIP

How to use the invNorm function on a TI-84 graphing calculator: Press 2nd VARS for the distribution menu, and then press 3 for invNorm. On your home screen, *invNorm (* will appear. Enter p, μ, σ)" so that the command looks like invNorm(p, μ, σ), and press ENTER.

Example D

To find the height of Dan Jennings, who is taller than 80% of the players in Major League Baseball, use the following command.

invNorm(0.8, 73.5, 2.25)

Answer: 75.39 inches

Try These D

a. Evaluate invNorm(0.65, 73.5, 2.25) on your calculator and interpret what each value represents in terms of Major League Baseball player heights.

b. Use your calculator to find the height of a player who is taller than 90% of all Major League Baseball players.

Lesson 36-4
Modeling with the Normal Distribution

My Notes

Answer the following questions in two ways. First, use the z-table method (include a sketch and shade a normal curve). Second, use technology with your graphing calculator. Recall that answers should agree very closely, but small rounding errors may cause them to be slightly different.

The distribution of batting averages for all Major League Baseball players very closely follows a normal distribution, with a mean of 0.261 and a standard deviation of 0.033.

1. A batting average of 0.300 or higher is considered very good. About what proportion of players have a batting average of at least 0.300?

2. One baseball player, Dewayne Wise, had a batting average that is in the first quartile of the batting average distribution. What was his batting average?

3. What range of batting averages gives the middle 50% of the distribution?

4. Miguel Cabrera of the Detroit Tigers had a batting average during the 2011 season of 0.344. What proportion of players had a batting average as high or higher than Miguel Cabrera during the 2011 season?

My Notes

Check Your Understanding

Normal distributions are associated with many populations that are not related to baseball. A wholesale nursery owner has 200 newly sprouted cocoplum plants that she is preparing for eventual sale. After several weeks, she measures each plant and discovers that the distribution of plant heights is approximately normal, with a mean of 8.5 cm and a standard deviation of 1.2 cm.

5. The nursery owner uses her graphing calculator and enters *normalcdf*(8, 9, 8.5, 1.2). What question is she seeking to answer with this calculation?

6. Cocoplum plants that are less than 6 cm tall are discarded, as they are unlikely to be sold. Use your graphing calculator to determine how many plants the nursery owner will discard.

7. Cocoplum plants that are larger than 10 cm are ready to be shipped for sale. Use your graphing calculator to determine how many plants are ready to be shipped.

LESSON 36-4 PRACTICE

The heights of 2-year-old American girls are distributed in an approximately normal manner. The 5th percentile and the 95th percentile of their heights are about 79 cm and 91 cm, respectively.

8. Estimate the mean and standard deviation of the distribution of 2-year-old girls' heights.

9. Use the mean and standard deviation to estimate the range of heights that would be in the middle 50% for 2-year-old girls.

10. About what proportion of 2-year-old girls are between 32 and 34 inches tall? (There are about 2.54 cm in an inch.)

11. Assume that from age 2 years to 5 years, all American girls grow 9 cm. How would this affect the mean and standard deviation of the population in Items 8–10?

ACTIVITY 36 PRACTICE

Write your answers on notebook paper.
Show your work.

1. Karen is a high school student doing a statistics project. She was interested in estimating how much money people typically spend on admission, food, drinks, and souvenirs when attending a local minor league baseball game. At one game she attended, she randomly selected 10 people in the audience and then asked them how much money they had spent. The responses are below.

$8.00 $10.25 $10.00 $9.50 $10.00

$10.25 $10.25 $12.75 $11.00 $11.25

a. Make a dot plot of these data.

b. These data are somewhat dense in the middle and sparser on the tails. Karen thought it would be reasonable to model the data as a normal distribution. She used the mean and standard deviation of her sample to estimate the mean and standard deviation of the amount of money spent by everyone at the ballgame that night. Based on her model, estimate the proportion of people attending the ballgame who spent between $10 and $12.

c. Again using Karen's model, estimate the amount of money that would complete this sentence: "95% of the people at the ballgame spent at least _______ dollars."

2. When students in Marty's statistics class were asked to collect some data of interest to them, Marty, a player on his school's baseball team, decided to measure the speeds of baseballs pitched by their school's pitching machine. Using a radar gun, he measured 20 pitches. The stem-and-leaf plot below shows the speeds he recorded, in miles per hour.

5	1 1 3
4	6 6 8 9 9 9
4	1 2 3 3 3 4 4 4
3	6 7 9

3 | 6 = 36 mph

a. Determine the mean and standard deviation of these 20 speeds.

b. Assuming that the distribution of speeds pitched by this machine is approximately normal, estimate how many pitches out of 100 you would expect to exceed 50 mph.

c. Assuming that the pitches from this machine are normally distributed, estimate the speed that would be at the 10th percentile of speeds pitched by this machine. What does the 10th percentile imply?

3. The annual salaries of nine randomly sampled professional baseball players, in thousands of dollars, are listed below.

1680, 316, 440, 316, 800, 347, 600, 16000, 445

a. If you assume that these come from a normal distribution, what proportion of all players would you expect to make over two million dollars (2000 thousands) per year?

b. What proportion of the nine players whose salaries are given have salaries over two million dollars per year?

c. You should have found that there is a pretty big discrepancy between your answers to Items 3a and 3b. Use what you know about normal distributions to explain this discrepancy.

d. Sketch a drawing of a normal distribution with the mean and standard deviation of these nine salaries. Comment on any features it has that may seem unrealistic.

4. Why is it important to look at a graphical display of a data set before performing probability computations that involve a z-table or a normal function on a calculator?

5. Performing normal computations directly on a calculator can be faster than using a z-table, but one potentially useful piece of information gets bypassed. What is it?

6. If you are using your calculator's built-in normal functions to answer questions without using the Standard Normal Table, sometimes you have to make up an upper or lower bound that wasn't stated in the question. When and why is that needed?

7. Below is a stem-and-leaf plot showing the distribution of ages, in years, of a random sample of 50 professional baseball players. The mean and standard deviation of the distribution are, respectively, 28.3 years and 5.1 years.

Stem	Leaf
4	2
4	1
3	8 9
3	7
3	5 5
3	2 2 2 3 3
3	0 0 0 0 1
2	8 8 8 8 9
2	6 6 6 6 7 7 7 7 7
2	4 4 4 4 4 4 4 4 5 5 5 5 5
2	2 2 3 3 3 3

2|2 represents 22

Would it be reasonable to use a normal distribution model to estimate the proportion of professional players who are 20 years old or younger? Explain your reasoning.

Use the following information for Items 8–10.
A math student who worked part-time at a veterinary clinic was given permission to examine the files of 11 adult cat patients and record their weights in pounds. These are the weights he recorded:

8.5, 9.1, 9.2, 10.2, 10.5, 11.1, 11.9, 11.9, 12.6, 13.6, 14.3.

8. Make a graph of the data to see whether it might be reasonable to believe that the distribution of weights of all cats at this clinic is approximately normally distributed. Comment on what feature(s) of the graph indicate that a normal model is or is not reasonable.

9. Assuming that a normal model is reasonable, about what fraction of cats at this clinic would weigh over 15 pounds?

MATHEMATICAL PRACTICES
Attend to Precision

10. Still using a normal model, estimate the range of weights that would be centered on the mean and encompass about 95% of cat weights.

Table A. Standard Normal Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

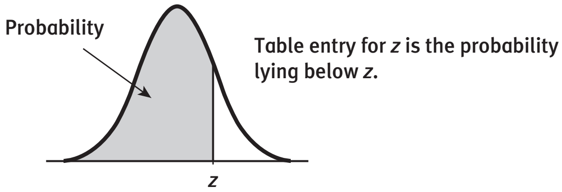


Table A. (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Random Sampling

Part-Time Jobs
Lesson 37-1 Surveys

My Notes

Learning Target:

- Explain why random sampling is advantageous when conducting a survey.

LEARNING STRATEGIES: Close Reading, Questioning the Text, Role Play, Summarizing, Paraphrasing, Debriefing, Discussion Groups

Jorge is a member of the student government at a large school with over 2500 students. The student government would like to recommend that students with part-time jobs be permitted to get a class credit in business. Knowing that Jorge is a good statistics student, the student government asked him to estimate the proportion of students at the school who have part-time jobs.

1. What difficulties might Jorge encounter if he tries to ask every student about having a part-time job?

Sometimes you may want to know some characteristic of a large population, such as the median income of households in your state or the proportion of students at a large school who have part-time jobs. Since it is often difficult or impossible to survey everyone in the population, you may wish to ***survey*** a ***sample*** of the population and infer conclusions from the sample about the population.

Jorge considers different methods for obtaining a sample.

2. Jorge is thinking about posting the question, "Do you have a part-time job?" on Facebook and collecting ***responses*** to his post. He knows that not everyone will reply, but he thinks he'll still get a large number of responses. Explain why, even if a large number of people replied (even as much as half of the student body), Jorge would be unwise to suppose that the proportion of people who posted that they have a part-time job is the same as the proportion of all students who have a part-time job.

MATH TERMS

A **survey** is a study in which subjects are asked a question or series of questions.

An answer provided by a subject to a survey question is called a **response**.

MATH TERMS

A **sample** is part of a population of interest. Data are collected from the individuals in the sample.

My Notes

3. Jorge is on the football team at his school and is thinking of asking everyone on the football team if they have a part-time job. Why might this give him a poor estimate of the actual proportion of students at his school with part-time jobs?

4. Jorge is considering standing beside an exit of the school one day after the last class is over and asking every student who passes by if he or she has a part-time job. How might this method produce an inaccurate estimate of the actual proportion of students at his school with part-time jobs?

MATH TERMS

A sample shows **bias** if the composition of the sample favors certain outcomes.

Sampling can give very good results even if only a small sample of the population is surveyed, but it is critical that the sample be *representative* of the population with respect to the survey question. If the design of a sample favors one outcome over another, the sample is said to be ***biased***. Each of Jorge's sampling methods described in Items 2, 3, and 4 display bias, and your responses indicate how this bias was manifested in the results.

How can you be sure that a sample is representative of the population? Many methods of sampling people could produce samples of people that would tend to favor one type of survey response over another.

MATH TERMS

A **simple random sample (SRS)** is a sample in which all members of a population have the same probability of being chosen for the sample.

One way to avoid favoring some types of response over others is to sample people at random, with every person being equally likely to be chosen. Such a sample is called a ***simple random sample***, abbreviated **SRS**. A simple random sample is impartial because it does not favor anyone over anyone else. When a simple random sampling process is used to select members from a population, then everyone is as likely to be included in the sample as everyone else, and one person's inclusion in the sample has no effect on anyone else's inclusion in the sample.

My Notes

5. There was bias in each of the sampling methods described in Items 2, 3, and 4 of this activity. Describe how a simple random sample would have avoided such bias.

6. Jorge has access to a full roster of all 2500 students at his school. One way to get a simple random sample of students would be for him to write the names of all 2500 students on index cards, put the cards into a large cardboard box and mix them up thoroughly, and then to draw out the desired number of names at random. What difficulties might Jorge encounter in his attempt to take a simple random sample in this way?

Another way to get a simple random sample is to number the list of students from 1 to 2500, and then use technology to randomly generate integers between 1 and 2500 until you have the desired sample size. For example, on TI-84 calculators, the following command generates a random integer between 1 and 2500:

randInt(1,2500)

Use the command to generate random integers that are matched up with the numbered list (ignoring repeated numbers) until you have identified all those names chosen to be in your sample.

TECHNOLOGY TIP

To find the *randInt(* function on a TI-84 calculator, press the MATH button, scroll to *PRB*, and then choose *randInt(*.

7. Use your graphing calculator to choose 20 random integers between 1 and 100. Write the calculator syntax and your 20 random integers.

My Notes

Another method for generating random numbers from 1 to 2500 involves using a random digits table. Since the largest number in this range has four digits, you need to represent all numbers from 1 to 2500 as four-digit numbers. For example, 23 would be represented as 0023, and 798 would be represented as 0798. Then choose a line of the table at random and begin inspecting clusters of four digits. When a four-digit number matches one on Jorge's list, that name is selected as part of the sample. If a number is not on the list, then it is disregarded, as are repeated occurrences of the same number.

Random digits

Line								
101	19223	95034	05756	28713	96409	12531	42544	82853
102	73676	47150	99400	01927	27754	42648	82425	36290
103	45467	71709	77558	00095	32863	29485	82226	90056
104	52711	38889	93074	60227	40011	85848	48767	52573
105	95592	94007	69971	91481	60779	53791	17297	59335
106	68417	35013	15529	72765	85089	57067	50211	47487
107	82739	57890	20807	47511	81676	55300	94383	14893
108	60940	72024	17868	24943	61790	90656	87964	18883
109	36009	19365	15412	39638	85453	46816	83485	41979
110	38448	48789	18338	24697	39364	42006	76688	08708
111	81486	69487	60513	09297	00412	71238	27649	39950
112	59636	88804	04634	71197	19352	73089	84898	45785
113	62568	70206	40325	03699	71080	22553	11486	11776
114	45149	32992	75730	66280	03819	56202	02938	70915
115	61041	77684	94322	24709	73698	14526	31893	32592
116	14459	26056	31424	80371	65103	62253	50490	61181
117	38167	98532	62183	70632	23417	26185	41448	75532
118	73190	32533	04470	29669	84407	90785	65956	86382
119	95857	07118	87664	92099	58806	66979	98624	84826
120	35476	55972	39421	65850	04266	35435	43742	11937
121	71487	09984	29077	14863	61683	47052	62224	51025
122	13873	81598	95052	90908	73592	75186	87136	95761
123	54580	81507	27102	56027	55892	33063	41842	81868
124	71035	09001	43367	49497	72719	96758	27611	91596
125	96746	12149	37823	71868	18442	35119	62103	39244
126	96927	19931	36089	74192	77567	88741	48409	41903
127	43909	99477	25330	64359	40085	16925	85117	36071
128	15689	14227	06565	14374	13352	49367	81982	87209
129	36759	58984	68288	22913	18638	54303	00795	08727
130	69051	64817	87174	09517	84534	06489	87201	97245

8. Beginning at line 122 on the random digit table, identify the first five numbers that would correspond to names on Jorge's list. Compare this method to using the random integer generator on the graphing calculator.

9. Suppose that Jorge uses the random number generator on his graphing calculator to choose an SRS of 100 students at his school. He then surveys these students to determine whether they have part-time jobs. He notices that two of the 100 students in his sample are friends who both have part-time jobs working at the local auto garage. Jorge is worried about the over-inclusion of people with part-time jobs in his sample. Should he be concerned?

My Notes

Check Your Understanding

10. Describe a sampling method that Jorge might have thought about using that would have likely *overestimated* the fraction of students at his school who hold part-time jobs.

11. Priscilla is a junior at the same high school. She would like to survey a simple random sample of the 600 juniors in her class to determine preferences for class T-shirt designs. Describe how she could create a SRS of 50 students using a random digits table and using a graphing calculator.

LESSON 37-1 PRACTICE

Veronica wanted to know how many students in the sophomore class at her school learned a language other than English as their first language. There were 450 sophomores in the sophomore class, too many for Veronica to question each of them, so she prepared 50 questionnaires to distribute to some of the students in the class.

12. In Veronica's survey, what is the population? What is the question of interest? What is the sample?

13. Veronica chooses two classes near her homeroom in which to distribute the questionnaires. One has 25 students and is for first-year Spanish learners, and the other has 25 students and is for ELL (English language learner) students. Why is this selection of students not a simple random sample? What type of bias may exist in this sample?

14. Describe how Veronica could create a simple random sample of 50 students from the sophomore class in two different manners, without using technology.

15. Describe how Veronica could use technology to create a simple random sample.

My Notes

Learning Target:

- Explain why random allocation of treatments is critical to a good experiment.

LEARNING STRATEGIES: Close Reading, Questioning the Text, Role Play, Summarizing, Paraphrasing, Debriefing, Discussion Groups

For a science fair project, Zack and Matt wanted to estimate how the rebound of a tennis ball is changed if it is soaked in water overnight and then allowed to dry out. They would have liked to get a random sample of tennis balls on which to perform an experiment, but they realized such a sample was impossible. Instead, their physical education coach gave them 20 used tennis balls as their sample.

MATH TERMS

An **experiment** applies a *treatment* (a condition administered) to *experimental units* to observe an effect.

The **explanatory variable** is what is thought to be the cause of different outcomes in the experiment. In simple experiments, the explanatory variable is simply the presence or absence of the treatment.

The effect of the explanatory variable is called the **response variable**.

1. Consider the definition of ***experiment***. Identify the ***explanatory*** and ***response variables***, the experimental units, and the treatment in Zack and Matt's experiment.

2. Why was it impossible for Zack and Matt to get a random sample of all tennis balls?

3. Zack and Matt decided to perform their rebound experiment on the 20 tennis balls their gym coach gave them. What limits on their conclusions would exist by performing the experiment with these balls?

Lesson 37-2
Experiments

My Notes

Zack and Matt planned to take their 20 tennis balls and put them into two groups of ten. The balls in one group would be soaked in water overnight and then allowed to dry out, while the others would just stay dry. They would then measure the rebound of all the tennis balls and compare the data for the two groups.

4. To determine which balls should be soaked and which would remain dry, Matt thought it best to use a ***completely randomized design***. Describe a process that would provide a completely randomized design for this experiment.

MATH TERMS

A **completely randomized design** implies that all experimental units have the same probability of being selected for application of the treatment.

5. Zack noticed that ten of the balls their coach gave them were Wilson brand balls, and the other ten were Dunlop brand balls. He thought that they should let the ten Wilson balls be the ones soaked in water and the ten Dunlop balls be the ones that stayed dry. What reasons might Matt have to disagree with Zack?

6. Matt suggested that it would be better to group all the Wilson balls and randomly choose five to be soaked in water. Similarly, he would group all the Dunlop balls and randomly choose five to be soaked in water. Why is this ***randomized block design*** a good strategy?

MATH TERMS

A **randomized block design** involves first grouping experimental units according to a common characteristic, and then using random assignment within each group.

My Notes

7. Matt and Zack thought about first measuring the rebound of all 20 dry tennis balls on a tennis court. Then they would soak all of the balls in water overnight and let them dry out. Finally, they would measure the rebound again on the tennis court. They could then see for each individual ball how much its rebound was changed by being soaked in water overnight. This strategy might be effective in accomplishing their research goal, but a critic of their experiment could point out that the change in rebound could be due to something other than having been soaked in water. Can you think of such a possible explanation?

MATH TERMS

A **matched pairs design** involves creating blocks that are pairs. In each pair, one unit is randomly assigned the treatment. Sometimes, both treatments may be applied, and the order of application is randomly assigned.

8. Describe how a ***matched pairs design*** may alleviate the potential problems identified in Item 6. Why would it be impossible to have matched pairs in which the order of treatment is randomized?

My Notes

Check Your Understanding

9. A random process was recommended to Jorge when he wanted to estimate how many students at his school hold part-time jobs. A random process was also recommended to Matt and Zack when they wanted to estimate the effect of waterlogging on tennis ball rebound. Explain how these two random processes are similar and how they are different.

LESSON 37-2 PRACTICE

A medical researcher wanted to determine the effect of a new drug on a specific type of cancer. He recruited 50 female and 50 male cancer patients, each diagnosed with this specific cancer that had progressed to the same stage. The anticipated effect of the drug was a 50% reduction in the size of the tumor within 4 weeks of treatment. All subjects would receive an injection, but some would receive the drug and others would receive a ***placebo***.

10. Describe a completely randomized experiment that the researcher could perform with these subjects.

11. Describe an experiment that would incorporate a block design and the purpose of the block design.

12. Describe an experiment that would incorporate a matched-pairs design and the purpose of the matched-pairs design.

13. A ***single-blind study*** is one in which either the person conducting the experiment or the subjects have knowledge of the treatment, but not both. A ***double-blind study*** is one in which neither the person conducting the experiment nor the subjects have knowledge of the treatment. Describe an advantage of a double-blind study in the cancer researcher's study.

ACADEMIC VOCABULARY

A ***placebo*** is a treatment applied to an experimental subject that appears to be the experimental treatment, but in fact is a treatment known to have no effect.

My Notes

Learning Target:

- Identify a confounding variable in an observational study.

LEARNING STRATEGIES: Close Reading, Questioning the Text, Role Play, Summarizing, Paraphrasing, Debriefing, Discussion Groups

Rebecca read an article online with the headline, "Survey shows that among employed Americans, people who text frequently tend to have lower-paying jobs than those who do not." Rebecca immediately sent a text message to her friend Sissy:

"OMG cc! txting makes u have less $$$! 2 bad 4 us!!!"

1. Why is the study referenced by the article that Rebecca read an ***observational study*** and not an experiment?

MATH TERMS

In an **observational study,** a researcher observes and records measurements of variables of interest but does not impose a treatment.

2. While it is possible that Rebecca is correct, the statement she read didn't say that texting caused people to have lower incomes, only that people who frequently text have lower incomes. Give another possible explanation for why those who text frequently may have lower-paying jobs.

MATH TIP

The results of an *observational study* can only imply an *association*. The results of an *experiment*, by imposing a condition, can imply *causation*.

If a study reports an association between two factors, and the researcher merely observed the association between the two variables without applying a treatment, then the researcher cannot determine if one of the factors directly caused the other. A third unmeasured variable that may be associated with both of the measured variables is called a ***confounding variable***. This variable is "confounded with" one of the other two, and therefore is a potential explanation of the association.

My Notes

3. A 2010 study reported that people who take long vacations tend to live longer than people who do not. One possible explanation is that vacations are good for you, improving your health and increasing your lifespan. Describe another potential explanation for the association, and identify a confounding variable.

A study published in the Journal of the American Medical Association showed that among a group of people who were hospitalized for bicycling accidents, the prevalence of elevated blood alcohol levels was significantly greater than it was among bicyclists who were stopped by the side of the road and who agreed to participate in the study by having their blood alcohol level measured.

4. Is there reason to believe that the actual proportion of (non-hospitalized) bicyclists who have elevated blood alcohol levels might be greater than what was estimated by recruiting bicyclists by the side of the road?

My Notes

5. The study included a caution about its conclusions, mentioning that the use of bicycle helmets was significantly more common among the people stopped by the side of the road than it was among those who were hospitalized. Why is that relevant to the conclusions one might draw from this study?

LESSON 37-3 PRACTICE

The crime rate in a small town was shown to be significantly higher whenever ice cream sales were higher. A town councilman was baffled by this, but nevertheless advocated closing down ice cream parlors to lower crime.

6. Identify the population and the question of interest in this study.
7. Was the ice cream crime rate study an experiment or an observational study? Explain your decision.
8. Write a letter to the councilman explaining why his position on closing ice cream parlors may be based on faulty reasoning. Include a potential confounding variable in your letter.

ACTIVITY 37 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 37-1

Following an online article about sunbathing posted on a website for teenagers, a poll asked the reader whether he or she regularly sunbathes. 81% of those who responded clicked on "Yes."

1. In this survey, what is question of interest?

2. What is the population that the survey seeks to represent?

3. What is the sample for this survey?

4. Is the sample representative of the population? Is it a simple random sample?

5. What bias may be apparent in the survey?

6. Describe how the bias in this survey may influence the results.

Lesson 37-2

A study was conducted to see whether drinking eight glasses of water daily would reduce the risk of catching a cold. Forty volunteers who participated in the study were randomly assigned to one of two groups. Those in one group were told not to change any aspect of their daily lives. Those in the other group were instructed to drink at least eight glasses of water daily. At the end of several months, the proportion of people who had caught a cold during that time period was significantly lower among those who drank at least eight glasses of water than among those who didn't. Since this was a randomized experiment, the researchers conducting the experiment thought that the only difference between the two groups of subjects was their water consumption, and, therefore, that drinking eight glasses of water daily can reduce your risk of getting a cold.

7. Why is this study an experiment as opposed to an observational study?

8. Describe a method that the researchers could have used to randomly assign members to each group.

9. What was the treatment in this experiment? What were the explanatory variable and the response variable?

10. Critics of the study identified something other than drinking water that made the two groups of subjects different from one another. What confounding variable may have influenced the results?

11. How could the experiment have been modified to eliminate the problem?

Lesson 37-3

For many years it was believed that playing classical music for infants was associated with these same people being smarter as older children and adults. Several early studies seemed to support this idea.

12. Valentina read one such study that claimed to be an observational study, not an experiment. Explain how such a study would be designed to be an observational study.

13. Identify a likely confounding variable in such a study, and explain how it could be responsible for the apparent association between listening to classical music and being smarter.

Bruno considered the classical music theory as well, but thought that an experiment would be better suited to test this theory.

14. For such an experiment, identify the question of interest, the experimental units, and the treatment.

15. With the help of a local daycare center, Bruno was able to identify 20 parents with infants between the age of 1 month and 2 months. Describe, in detail, an experiment that would test the question of interest.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

16. Suppose Bruno's experiment reveals a significant increase in intelligence for those children who listened to classical music. What limitations may exist in the interpretation of the results?

Normal Models, Surveys, and Experiments
RESEARCHING READERS

1. A researcher in psychology measured the reading skill, on a scale of 1 to 100, of a random sample of 16 fifth-graders at a school. The skill levels were as follows:

51	82	65	69	69	71	58	72
68	76	56	61	77	64	63	71

Assume that it is reasonable to model the distribution of reading skill levels of all fifth-graders at the school as approximately normal.

a. Estimate the proportion of fifth-graders at the school with reading skill levels at or below 55.

b. Estimate the proportion of fifth-graders at the school with reading skill levels between 60 and 70.

c. Estimate the reading skill level that a fifth-grader would have if his or her score was in the 95th percentile of reading skill levels for fifth-graders at the school.

d. Create a data display and explain how it supports or conflicts with the assumption of an approximately normal distribution for this data set.

2. A study was done in which volunteer subjects were divided into two groups at random. Subjects in the first group read realistic news stories about fictitious politicians and their political activities. Subjects in the second group read the same stories, but they also read stories about scandals involving the politicians. After several weeks, the subjects were asked to recall information about the politicians. The subjects in the second group recalled more about the activities of the politicians than did the subjects in the first group.

a. Identify the treatment, explanatory variable, and response variable in this experiment.

b. What might the researchers conclude as a result of this study?

c. Suppose that researchers used a block design in the experiment, placing subjects who regularly read news stories in one group and those who did not regularly read news stories in another group. Explain how this may have changed the conclusions that could be drawn from this study.

3. An online survey on a vegetable gardening website found that respondents who planted after April 1 had greater yields than those who planted before April 1.

a. Describe why this survey is an example of an observational study and not an experiment.

b. Brianna read the survey results and commented, "Planting after April 1 must cause vegetables yields to be greater." Describe the flaw in her statement.

c. Why might someone be skeptical about the results of such a survey?

Normal Models, Surveys, and Experiments
RESEARCHING READERS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3)	• Clear and accurate understanding of statistical concepts including survey, observational studies, and experimental design, and the impact of randomization on each • Clear and accurate understanding of population means and proportions, percentiles, and properties of a normal distribution	• A functional understanding and accurate interpretation of statistical concepts including survey, observational studies, and experimental design, and the impact of randomization on each • A functional and mostly accurate understanding of population means and proportions, percentiles, and properties of a normal distribution	• Partial understanding and partially accurate interpretation of statistical concepts including survey, observational studies, and experimental design, and the impact of randomization on each • Partial understanding and partially accurate work with population means and proportions, percentiles, and properties of a normal distribution	• Little or no understanding and inaccurate interpretation of statistical concepts including survey, observational studies, and experimental design, and the impact of randomization on each • Little or no understanding and inaccurate work with population means and proportions, percentiles, and properties of a normal distribution
Problem Solving (Items 2, 3)	• An appropriate and efficient strategy that results in a correct answer	• A strategy that may include unnecessary steps but results in a correct answer	• A strategy that results in some incorrect answers	• No clear strategy when solving problems
Mathematical Modeling / Representations (Item 2)	• Clear and accurate understanding of how to apply experimental design models to a real-world scenario	• Mostly accurate understanding of how to apply experimental design models to a real-world scenario	• Partial understanding of how to apply experimental design models to a real-world scenario	• Inaccurate or incomplete understanding of how to apply experimental design models to a real-world scenario
Reasoning and Communication (Items 2, 3)	• Precise use of appropriate math terms and language to describe the differences between observational studies and randomized experiments and justify reasoning regarding statistical models • Clear and accurate explanation of the effects of changing conditions in a study and why results may not be valid	• Adequate description of differences between observational studies and randomized experiments and justification of reasoning regarding statistical models • Adequate explanation of the effects of changing conditions in a study and why results may not be valid	• Misleading or confusing description of differences between observational studies and randomized experiments and justification of reasoning regarding statistical models • Misleading or confusing explanation of the effects of changing conditions in a study and why results may not be valid	• Incomplete or inaccurate description of differences between observational studies and randomized experiments and justify reasoning regarding statistical models • Incomplete or inadequate explanation of the effects of changing conditions in a study and why results may not be valid

Simulations

Is Martin Improving?

Lesson 38-1 Devising Simulations

My Notes

Learning Target:

- Devise a simulation that can help determine whether observed data are consistent or inconsistent with a conjecture about how the data were generated.

SUGGESTED LEARNING STRATEGIES: Close Reading, Predict and Confirm, Summarizing, Paraphrasing, Think Aloud, Debriefing, Discussion Groups

Martin enjoys playing video games. On his birthday he received "Man vs. Monsters," a game in which the player plays the role of a person who is trying to save the earth from an invasion of alien monsters. At the end of the game, the player either wins or loses. The first three times Martin played the game, he lost. In the next seven games that he played, he won four times, and he felt like his performance was improving. In fact, the sequence of Martin's wins and losses is as follows, where "L" represents losing a game, and "W" represents winning a game.

L, L, L, W, L, L, W, L, W, W

Martin concluded he was getting better at the game the more he played, and he said that this sequence of wins and losses was evidence of his improvement. His sister Hannah, however, was not convinced. She said, "That sequence of wins and losses looks like a random list to me. If you were really getting better, why didn't you lose the first six and then win the last four?"

In this activity, you will use a ***simulation*** to decide who is correct, Martin or Hannah.

MATH TERMS

A simulation is a process to generate imaginary data, often many times, using a model of a real-world situation.

Start by considering that Hannah is correct and that Martin was not really getting better. He had six losses and four wins in a particular order and, if Hannah is correct, those wins and losses could have been arranged in any other order. According to Hannah, Martin's results indicate how good he is at the game—he wins about 40% of the time—but do not indicate whether he is improving.

1. Following this page are ten squares, six of which are marked "Lose" and four of which are marked "Win." These represent the outcomes of the ten games Martin played. Cut out the squares and arrange them *facedown* on your desk.

My Notes

2. Once you have placed the cards facedown, mix them up and arrange them in a random sequential order so that you do not know which ones represent wins and which ones represent losses. Then turn them all face up so you can see the L or W, and write down the order of wins and losses here. This is a simulation of Martin's wins and losses.

3. Consider the following two sequences, and write a sentence explaining whether it appears that Martin is improving.

a. L, L, L, L, W, L, L, W, W, W

b. W, L, W, W, L, L, L, W, L, L

4. It is desirable to quantify (i.e., measure with a numerical quantity) the extent to which a sequence of wins and losses indicates that a player who achieved it is really improving. Describe a method that may quantify the results of playing ten games such that the number describes the improvement of a player. Be creative!

My Notes

Lose	Lose
Lose	Lose
Lose	Lose
Win	Win
Win	Win

My Notes

This page is intentionally blank.

My Notes

One way to quantify improvement is to count how many wins occur among the last five games, and subtract the number of wins that occur among the first five games. Call this the "improvement score" for the sequence of results for ten games.

5. Two example sequences are provided below. Find the improvement score for each. Show all work.

L, L, L, L, W, L, L, W, W, W

W, L, L, W, L, L, L, W, L, W

6. Using this method to quantify improvement, what would a negative improvement score imply? What would a positive improvement score imply?

Any number that summarizes data in a meaningful way is called a ***statistic***. Your improvement score, a number which is the difference between the number of wins among the last five of ten games and the number of wins among the first five, is a statistic because it summarizes the data with a number that measures improvement.

7. Compute the improvement score for the sequence of wins and losses from your simulation in Item 2 when you mixed up the order of your ten squares.

MATH TERMS

A **statistic** is a number that summarizes data in a meaningful way. The mean of a data set is an example of a statistic.

My Notes

Recall the reason for computing the improvement score. Martin's sister, Hannah, is skeptical that Martin's ability to win the game is improving. She thinks that his particular sequence of wins and losses looks random and does not imply improvement. To address her concern, it is important to determine whether a sequence like Martin's might easily show up if the order of wins and losses really is random. More specifically, it is important to determine if the improvement score that results from Martin's sequence of wins and losses is a number that might easily result from a random arrangement of four wins and six losses.

8. Compute the improvement score for Martin's actual sequence of wins and losses:

L, L, L, W, L, L, W, L, W, W

Check Your Understanding

In Item 4, you created a statistic to measure improvement. Below are two other possible "improvement statistics" that Martin might have used to measure his improvement over ten games. For each one, state (a) whether the statistic is actually a measurement of improvement and (b) whether the statistic is likely to provide more information than Martin's improvement score as defined before Item 5. Explain your answers briefly.

9. Count the number of games until Martin achieves his second win. This number of games is the improvement statistic.

10. Identify each win with a "1" and each loss with a "0". Create ordered pairs such that the number of the game (1 through 10) is the x-coordinate and the "1" or "0" is the y-coordinate. Use technology to make a scatter plot of these ten points and compute the slope of the regression line through the ten points. The slope of the regression line is the improvement statistic. Write the linear equation of the regression line.

My Notes

LESSON 38-1 PRACTICE

Teresa conducted a survey of a simple random sample of ten customers shopping in a grocery store. Her survey asked the customers to identify the price of the most expensive item in their basket. The ten responses, rounded to the nearest dollar, are listed below.

12, 8, 3, 2, 9, 25, 14, 8, 4, 5

11. Identify two statistics that could be calculated from these data.

12. Calculate the statistics that you identified in Item 11, and describe the significance of each statistic.

Steven would like to create simulations that would model the incidence of precipitation in a particular city.

13. Consider a fictional city where data indicate that precipitation occurs on 50% of the days in a year. Describe how Steven could perform a simulation to determine the occurrences of precipitation in this city during eight randomly chosen days of the year, using a fair coin.

14. Sacramento, California, receives rain on approximately one of every six days during a year. Describe a method by which Steven may simulate precipitation in Sacramento for eight randomly chosen days of the year.

15. Vero Beach, Florida, receives rain on approximately one of every three days during a year. Describe a method by which Steven may simulate precipitation in Vero Beach for eight randomly chosen days of the year.

16. Hilo, Hawaii receives rain on approximately three of every four days during a year. Describe a method by which Steven may simulate precipitation in Hilo for eight randomly chosen days of the year.

My Notes

Learning Target:

- Determine if a simulation indicates whether observed data are consistent or inconsistent with a conjecture about the data.

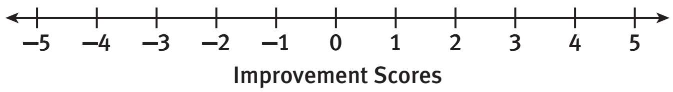
SUGGESTED LEARNING STRATEGIES: Close Reading, Predict and Confirm, Summarizing, Paraphrasing, Think Aloud, Debriefing, Discussion Groups

1. In the previous lesson, you carried out a simulation by mixing ten cards representing Martin's wins and losses. Next you created a sequence of the results and then computed the improvement score for the sequence you created. Repeat that process, recording below the improvement score for each randomly ordered sequence of wins and losses that you get. Work with your group and collect your results together until you have collected 40 improvement scores. (Keep all 40 sequences for use later in this activity.)

Lesson 38-2
Confirming Data with Simulations

My Notes

2. Make a dot plot showing the distribution of the improvement scores that you found in Item 1.



3. Recall Martin's improvement score that you found in Item 7 of Lesson 1. Describe the column of dots in your dot plot that corresponds to Martin's improvement score.

4. Why are there no improvement scores of ± 1, ± 3, ± 5?

5. Based upon your results, what is the probability of Martin obtaining the improvement score that he received in his initial game? What is the probability of receiving *at least* that score?

My Notes

6. Is Martin's improvement score one that is likely to occur by chance?

7. Consider the event that Martin's sequence of game results was LLLLLLWWWW. Determine his improvement score for this game, and interpret the score with respect to Hannah's claim that his results did not indicate improvement.

Check Your Understanding

8. A physical education class with 15 female students and 10 male students had to select 11 students at random to form a soccer team. Bob was skeptical when the teacher announced that all 11 players selected were female. Describe a simulation that Bob could perform that would determine if such a selection was likely a result of chance or a result of some bias.

9. One method of proof in mathematics is known as "proof by contradiction." In such proofs, you begin with a negation of the statement you wish to prove. Then, through logical deduction using known facts, a false statement is concluded. Since the conclusion is false, the original statement must be false, and the statement you want to prove is correct.

Identify one similarity and one difference between a mathematical proof by contradiction and the logical argument that you made in Items 6 and 7.

My Notes

LESSON 38-2 PRACTICE

Consider the following alternative statistic to measure improvement: add together the position numbers of all the wins. The larger the total is, the later in the sequence the wins must be. For example:

$$\text{L, L, L, L, W, L, L, W, W, W} \rightarrow 5 + 8 + 9 + 10 = \mathbf{32}$$

$$\text{W, L, L, L, L, W, W, W, L, L} \rightarrow 1 + 6 + 7 + 8 = \mathbf{22}$$

Call this new statistic the "improvement measure." In the items that follow, use the improvement measure to see whether Martin's particular sequence of wins and losses could easily be explained by his sister Hannah's theory that his wins and losses were really just in a random order.

10. Determine Martin's improvement measure.

11. Describe how you will simulate whether or not Martin's sequence of game outcomes is consistent with Hannah's theory.

12. Show the distribution of the improvement measures that result from many random orderings of Martin's game outcomes. Use the sequences you obtained from the 40 trials in Item 1 of this lesson.

13. State a conclusion about whether Martin's sequence of wins and losses is consistent with Hannah's theory.

14. Explain the logic that led you to your conclusion.

CONNECT TO AP

In AP Statistics, it is critical that students be able to write coherent and clear descriptions of simulations that even a non-statistician would be able to follow.

ACTIVITY 38 PRACTICE

Write your answers on notebook paper.
Show your work.

Use the following information for Items 1–5.

Jesse, a high school junior, was talking with six of his friends about whom they planned to vote for in the upcoming election of the class president. There were two candidates, Sarah and John. Among Jesse's group of friends there were three girls, and all of them planned to vote for Sarah, a girl. Jesse's three other friends were boys, and two of them planned to vote for John, a boy. Only one friend of Jesse's—a boy—was planning to vote "against gender" and vote for Sarah. Jesse thought that his friends were voting according to their own gender and wondered if this was just a chance occurrence.

1. Jesse wants to perform a simulation to determine if his friends' tendency to vote according to gender was likely a result of random chance. Describe (but do not perform) a simulation that Jesse could perform to accomplish this task.
2. Identify a statistic that Jesse could measure in his simulation.
3. Describe the process for determining the likelihood of the occurrence of the statistic for Jesse's friends.
4. Based on your results from Item 3, assume that the probability of the occurrence of the statistic was 0.40. What conclusion would you make?
5. Based on your results from Item 3, assume that the probability of the occurrence of the statistic was 0.05. What conclusion would you make?

Use the following information for Items 6–9.

For a research project, Tia wanted to see whether people could tell the difference between two brands of cola by taste. She planned an experiment. Volunteer subjects would each be presented with three small identical-looking cups of soda labeled A, B, and C. Two of the cups would contain the same brand of cola while the third cup would contain the other brand. Tia would randomly determine which of the three cups would be the one containing the different brand. She would also randomly determine which cola brand would be in two cups and which would be in one cup.

Each subject would be asked to taste the cola in each cup and then identify which cup contained the different brand. The subjects would not be required to identify the brands, only to tell which cup contained a different brand.

After getting responses from 20 subjects, Tia planned to count how many had identified the correct cup, and then see whether that count was too large to be explainable by just random chance.

6. Identify the statistic that Tia is measuring.
7. Tia is interested in seeing whether her statistic is greater than she would expect by chance alone. What would the value of her statistic be if no one could taste a difference between the two drinks?

Use the following information for Items 8 and 9.

Suppose that 12 of the 20 people in Tia's experiment gave correct cup identifications. Describe a process by which Tia could decide whether 12 correct cup identifications would or would not be surprising if, in fact, everyone was just guessing.

8. Describe such a process using a six-sided number cube. Be sure to identify what each roll of the number cube represents and what the numbers on the number cube represent. You do not have to carry out the process—just describe it clearly.

MATHEMATICAL PRACTICES
Make Sense of Problems and Persevere in Solving Them

9. Describe another such process using only a random number table. Be sure to identify what each digit represents and the meaning of that digit.

Margin of Error

Can't Get No Satisfaction

Lesson 39-1 Introduction to Margin of Error

ACTIVITY 39

My Notes

Learning Targets:

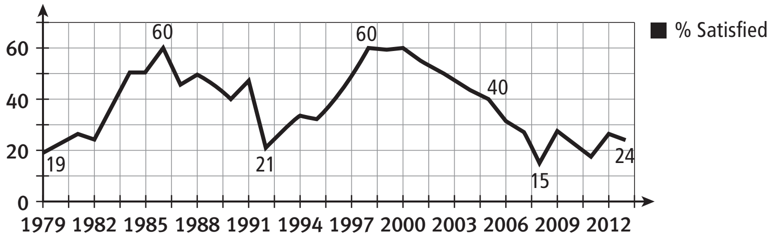
- Use margin of error in an estimate of a population proportion.
- Use simulation models for random samples.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think Aloud, Debriefing, Discussion Groups

Since 1979, Gallup, a national polling organization, has reported survey results of the question, "In general, are you satisfied or dissatisfied with the way things are going in the United States at this time?"

The results from 1979 to 2012 are displayed on the graph shown.

Satisfaction With the Way Things Are Going in the U.S., Yearly Averages



2013 figure represents yearly average to date.

1. Describe the meaning of the graph and characteristics that may be of interest to a person studying this graph.

2. From 2000 to 2012, there is a steady decline in the satisfaction proportion. What historical events may account for such a decline?

My Notes

The results of the 2013 Gallup poll asking this question, conducted on November 7–10, 2013, indicated that 20% of Americans are satisfied with the way things are going in the United States. These results were based on telephone interviews with a random sample of 1039 adults, aged 18 and older, living in all 50 U.S. states and the District of Columbia.

3. Why did the Gallup pollsters use a random sample to establish this proportion of satisfied Americans?

Random samples are frequently used to make inferences about entire populations. Since the samples chosen are random and rely on chance, the laws of probability allow us to determine how sample results compare to an actual population proportion. The Gallup poll description continues with the following statement: "One can say with 95% confidence that the margin of sampling error is ± 4 percentage points."

4. What is the meaning of this statement with respect to the fact that 20% of the Americans polled stated that they were satisfied with the way things were going in the United States?

MATH TERMS

The **margin of error** indicates how close the actual proportion is to the estimate of the proportion found in a survey of a random sample.

The phrase "± 4 percentage points" in the statement is called the ***margin of error***. Random samples have characteristics that set bounds on the errors that are likely to exist in the results of that random sample. In this activity, we will investigate these characteristics.

5. The Gallup poll indicated that 20% of the population was satisfied with how things were going in the United States in November 2013. If the actual population proportion is 20%, how many satisfied people would you expect from a random sample of ten people?

My Notes

6. It is possible that your random sample of ten people in Item 5 could yield results that differ from your answer to Item 5. Which results would not be surprising? Which results would be surprising?

7. Given the actual population proportion is 20%, how many satisfied people would you expect from a random sample of 100 people? How different from your expected value must a result be for it to be a "surprising" result?

MATH TIP

To perform a simulation of a survey, generate imaginary data based on assumptions about actual population characteristics.

Using your graphing calculator, you can perform a simulation for the situations in Items 5 and 7 to model the selection of a random sample and the number of "successes" in that sample.

8. Use the *randBin(* function of your calculator to perform ten different simulations of the survey in Item 5. How many satisfied people exist in a random sample of ten people if the actual proportion is 0.20? Does your result agree with your answer to Item 6?

TECHNOLOGY TIP

To find the *randBin* function on the TI-84, press MATH and the arrow keys to select the *PRB* menu, and select *randBin(*. The first entry is the number of subjects in the random sample, followed by a comma, and then the probability of "success" for each subject in that random sample. Press ENTER and the result is the number of "successes" for one random sample. If you would like to perform the simulation a number of times, you can follow the probability with a comma, followed by the number of simulations you would like to perform.

For example, to find the number of successes in one random sample of ten people with a probability of success of 0.5, enter *randBin(10, 0.5)* To find the number of successes in eight such random samples, enter *randBin(10, 0.5, 8)*.

9. Compare the result of your imaginary survey with the ones conducted by the others in your group. Explain why the results are likely different from one another.

My Notes

10. Since the survey results are concerned with the proportion of people who are satisfied, convert each of your results into a *proportion*. The proportion for each result is called the ***sample proportion***. Combine the proportions from your surveys with the others in your group so that you have 40 survey results.

a. Create a histogram to display the distribution of proportions, and comment on the shape of your group's distribution.

b. Compute the mean and standard deviation for the 40 survey proportions.

11. Use the *randBin(* function of your calculator to perform ten different simulations of the survey in Item 7. How many satisfied people exist in a random sample of 100 people if the actual proportion is 0.20? Does your result agree with your answer to Item 7?

My Notes

12. Combine the results of your survey with the others in your group so that you have 40 survey results, and find the *proportion* of satisfied subjects for each survey.

a. Create a histogram to display the distribution of proportions, and comment on the shape of your group's distribution.

b. Compute the mean and standard deviation for the 40 survey proportions.

13. Compare and contrast the means and standard deviations for the combined surveys of ten subjects and for the combined surveys of 100 subjects. What conclusion can you infer from these results?

My Notes

Check Your Understanding

14. In the days prior to a mayoral election, a poll reported, with 90% confidence, that the current mayor had support of 53% of the city's voting population, with a margin of error of 6%. Write a sentence to interpret the results of the survey.

15. Describe a procedure that uses a number cube to simulate a population proportion of 33%. How many successes would you expect from 12 trials? Perform the simulation 12 times, record your results, and compare them to your expectations.

LESSON 39-1 PRACTICE

16. Jorge claimed that the results of a survey supported his claim that most of the students in the junior class scored above average on the PSAT test. Valentina read the results of the survey to Jorge: "A survey of a simple random sample of students in the junior class indicated that 48% of them scored above average on the PSAT test. One can say with 95% confidence that the margin of error for this survey is plus or minus 4%." Is Jorge correct that the survey supported his claim?

The Gallup-Healthways Well-Being Index tracks, on a daily basis, the proportion of Americans who say they experienced happiness and enjoyment without stress and worry on the previous day. On one particular day, the survey of 500 people indicated that 54% were happy, with a margin of error of $\pm$ 5%.

17. Using technology or a random digits table, describe how you could simulate 20 repetitions of such a survey for a random sample of size 100.

18. Perform the simulation that you described in Item 15, and find the mean and standard deviation.

19. Change your results to proportions and display them on a histogram. Use an interval width of 0.1.

20. Describe the shape of your distribution. Identify proportions that you would expect in such a simulation, and identify proportions that would be surprising in such a simulation.

My Notes

Learning Targets:

- Use margin of error in an estimate of a population proportion.
- Relate margin of error to the population proportion and to the sample size.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think Aloud, Debriefing, Discussion Groups

"In general, are you satisfied or dissatisfied with the way things are going in the United States at this time?" For this question of interest, recall that the Gallup organization reported that for results based on this sample of 1039 adults, you can say with 95% confidence that the margin of error is ± 4 percentage points.

The distribution of proportions of those who indicate they are satisfied for all possible samples of size n from the population is called the ***sampling distribution*** of the population for that statistic.

1. What is the population for this question of interest? Why is it not feasible to find the sampling distribution of size $n = 1039$ for this population?

While it is not possible to find the sampling distribution for this statistic, you did generate some ideas by finding a large number of samples using simulations in the previous lesson.

2. In Items 10 and 12 from Lesson 39-1, which distribution was approximately normal? What were the sample sizes in those distributions?

My Notes

As sample sizes increase, the sampling distribution becomes more and more normal. If a random sample of size n has a proportion of successes p, there are two conditions that, if satisfied, allow the distribution to be considered approximately normal. Those two conditions are $\boldsymbol{n(p) \geq 10}$ and $\boldsymbol{n(1 - p) \geq 10}$.

3. Show that Gallup's survey meets the normal conditions.

4. Show that the simulation performed with $n = 10$ *does not meet* the normal condition and that the simulation performed with $n = 100$ *does meet* the normal condition.

Lesson 39-2
Computing Margin of Error

In general, when investigating a question of interest, you are not aware of the actual population statistic. However, by taking a simple random sample of an appropriate size, you can make inferences about the entire population. Also recall that normal distributions are completely described by two statistics: the mean and the standard deviation.

The standard deviation for a sampling distribution is given by

$$\sqrt{\frac{p(1-p)}{n}}$$

5. What is the meaning of the standard deviation with respect to a sample proportion?

6. In your simulations, you used $p = 0.20$ and $n = 100$. To be more accurate, would you prefer to use $n = 1000$? Use the formula to evaluate standard deviations to support your answer.

My Notes

MATH TIP

In the previous lesson, you discovered that the mean of the proportions of your sampling distributions was very close to the actual proportion. This is because the mean of the proportions of the entire sampling distribution is equal to the actual proportion. Therefore, *we can consider the proportion p of the random sample as the actual proportion.*

My Notes

MATH TERMS

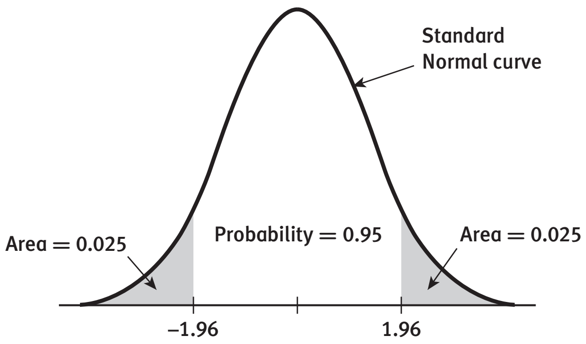
A **critical value** for an approximately normal distribution is the *z*-score that corresponds to a level of confidence.

The Gallup survey stated that "the margin of error is ±4 percentage points." The margin of error is the range about the sample proportion in which you would expect to find the actual population proportion. The margin of error is found by multiplying the standard deviation by the ***critical value***.

Example A

A city government said that, based on a survey of a random sample of 800 adults in the city, you can say that 25% of them prefer weekly recycling pickup, with 95% confidence that the margin of error is ±3 percentage points.

- The sample proportion is 0.25.
- Since $np > 10$, $800(0.25) = 200 > 10$ and $n(1 - p) > 10$, $800(1 - 0.25) = 600 > 10$, we can assume that the sampling distribution is approximately normal.
- You would like to be 95% confident in the statement; this will determine the critical value. Since the distribution is approximately normal, we can use the *z*-table or invNorm function on our calculators. Notice that the 95% interval is evenly divided on either side of our sample proportion (mean).



- Find 0.975 in the body of the *z*-table for the positive critical value (1.96) or 0.025 in the body of the table for the negative critical value (−1.96).
- Multiply ± 1.96 by the standard deviation,

$$\pm 1.96\sqrt{\frac{0.25(1-0.25)}{800}} \approx \pm 1.96(0.0153) \approx 0.030$$

- 0.030 is the margin of error.
- Therefore, you are 95% confident that the actual proportion of city residents that prefer weekly recycling pickup is 25% with a margin of error of ± 3%.

TECHNOLOGY TIP

You may also use *invNorm*(0.25,0,1) on the TI- 84 to find the critical value. Use the mean 0 and standard deviation of 1 in this function because you are assuming that the values are standardized.

Lesson 39-2
Computing Margin of Error

My Notes

Victor, a member of the Student Government Association at his high school, wanted to know if students approved of the theme of the school's homecoming dance. He polled a simple random sample of 120 subjects from the population of 2000 students at his school, and 72 of the responses indicated approval. Victor would like to report back to the SGA with 90% confidence in the results of his survey.

7. What is the sample proportion that indicated approval?

8. Victor assumes that the sampling distribution for his poll is approximately normal. Show that he is correct in his assumption.

9. Victor wants to report with 90% confidence in his results.

a. On a normal distribution with 90% evenly divided on either side of the sample proportion (mean), what two probability values would you want to identify?

b. What are the critical values associated with these probabilities?

10. What is Victor's margin of error?

My Notes

11. Write a sentence that reports Victor's results to the Student Government Association at his school.

12. Without performing the computations, how do you think the margin of error would change if the number of students that Victor polled were 80? How do you think it would change if the number of students were 200?

13. Compute the actual margin of error for $n = 80$ and $n = 200$ to confirm or revise your answer to Item 7.

My Notes

Check Your Understanding

Recall that the standard deviation of a sample proportion is represented by $\sqrt{\frac{p(1-p)}{n}}$.

14. Describe the meaning of each variable. Explain what happens to the standard deviation when the value of n increases.

15. For a fixed value of n, what value of p would yield the largest standard deviation?

LESSON 39-2 PRACTICE

Sofia is a credit card specialist with a large financial institution. She is interested in knowing what proportion of the bank's credit card holders have credit scores in the good or excellent range (scores of 680 and above). Sofia surveyed a simple random sample of 1000 of the bank's credit card customers and found that 750 of them had credit scores of 680 and above.

16. For Sofia's survey, identify each of the following.
- **a.** the question of interest
- **b.** the population
- **c.** the sample proportion

17. Write the standard deviation for the sample proportion.

18. Sofia wants to be 98% confident in her estimate of the actual proportion. What critical values will she use in her determination of the margin of error?

19. Compute the margin of error, and write a sentence that describes the results of Sofia's survey.

ACTIVITY 39 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 39-1

A jar contains 1000 jellybeans that are colored either red or green. 30% of the jellybeans are red, and the remaining jellybeans are green. Assume that the jellybeans are well mixed and that a random sample of 20 jellybeans is chosen from the jar.

1. Would it be unusual to pull out five red jellybeans and 15 green jellybeans? Explain.
2. Would it be unusual to pull out 15 red jellybeans and five green jellybeans? Explain.
3. Describe a simulation that you could perform with a random digits table to model 30 such samples.
4. Describe a simulation that you could perform with a graphing calculator that would model 30 such samples.
5. Perform one of the simulations that you described in Item 3 and Item 4. Convert the number of jellybeans to proportions, and explain how the results of your simulation agree or disagree with your responses to Item 1 and Item 2.

Lesson 39-2

In late 2011 the Gallup organization surveyed a random sample of 2007 American adults and asked them what they thought about China's relationship with the United States. 76% of those surveyed said that China was either "friendly" or "an ally." Gallup reported the following statement along with the survey results: "For results based on the total sample size of 2007 adults, one can say with 95% confidence that the margin of error attributable to sampling and other random effects is ± 2.68 percentage points."

6. What is the population?
7. What is the question of interest?
8. What is the sample proportion?
9. The margin of error reported uses some advanced statistical methods to adjust for sample and population characteristics. Find the "unadjusted" margin of error for this survey.
10. If the sample size for this survey were 1000, what changes would you expect in the margin of error?
11. Compute the margin of error for a sample size of 1000.

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

12. In 2011, a New York University professor of journalism, Charles Seife, wrote, "Random events behave predictably in aggregate even if they're not predictable individually." How does that principle relate to the concept of a margin of error in a survey result?

Designing and Conducting Simulations

Time Flies When You Are Having Fun

Lesson 40-1 Random Chance

My Notes

Learning Target:

- Determine whether an apparent treatment effect is too large to be due just to random chance.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, KWL Chart, Role Play, Summarizing, Paraphrasing, Think Aloud, Debriefing

As you read the scenarios and problems in this activity, mark the text to identify key information and parts of sentences that help you make meaning from the text.

Jamie and Riley wanted to see whether the adage "Time flies when you are having fun!" could be demonstrated scientifically. They decided to conduct a study and recruited 14 classmates to be subjects. Jamie randomly selected seven subjects and assigned them to a group called "Fun." The rest were assigned to a group called "Not Fun." The "Fun" group was given a task of playing a video game enjoyed by all subjects, while the "Not Fun" group was assigned a task of copying code for a programming language with which none were familiar.

1. Is the study described an observational study or an experiment? Explain your reasoning.

Each subject was asked to spend 30 minutes in a quiet room performing their assigned task with no time-keeping capability. At exactly 13.5 minutes into their task, subjects were interrupted and asked to estimate the number of minutes that had passed since the task began.

2. What are the variables in this study?

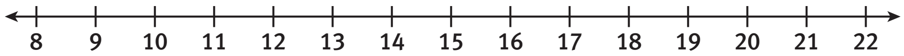
My Notes

After completing the study for all 14 subjects, Jamie and Riley wanted to analyze the data to determine whether the subjects in the "Fun" group tended to think less time had passed than those in the "Not Fun" group. If they did, then there would be evidence that the expression, "Time flies when you are having fun!" is true.

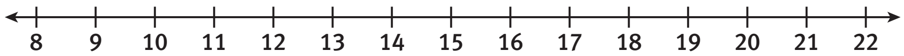
The table below gives Jamie and Riley's data.

Group	Perceived Minutes Elapsed						
Fun	10	11	10	15	9	14	14
Not Fun	18	17	17	15	10	12	20

3. Draw dot plots of the data on the axes below to display the distributions of perceived elapsed times.



Fun



Not Fun

4. Compare the two distributions of estimated elapsed times in a way that addresses Jamie and Riley's research question.

My Notes

Jamie and Riley showed their data to their classmate, Mercedes, who raised an issue they had not considered. She asked, "How do you know that your subjects would not have produced the same results regardless of which group they were in?"

5. Suppose that the perception of the passage of time was not affected by the group in which the subjects were placed. Describe how the two distributions would have appeared.

Mercedes suggested that Jamie and Riley consider the difference between medians of the two sets of data—median of the "Not Fun" group minus median of the "Fun" group—to compare the two groups.

6. Compute the difference in medians of the two sets. Does this indicate a difference between the two groups?

7. How large must the difference between the medians be to show an effect of the task assigned to each group?

My Notes

MATH TERMS

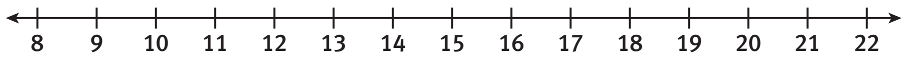
A statistic computed from a set of data is **statistically significant** if it would have been very unusual for the value of the statistic to be the result of chance alone.

As a result of chance, two data sets cannot be expected to have exactly the same values, even if the treatments really had no effect. However, if the difference in two data sets is so great that it is very unlikely to have occurred by chance, then the results are called ***statistically significant***.

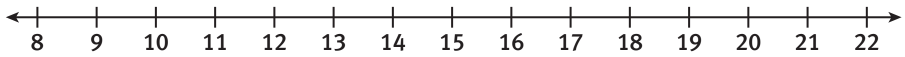
8. Using the data that Jamie and Riley collected, rearrange the "Perceived Minutes Elapsed" values in such a manner that they would be most statistically significant.

Group	Perceived Minutes Elapsed						
Fun							
Not Fun							

9. Create a dot plot for these groups, find the median for each, and compute the difference in medians.



Fun

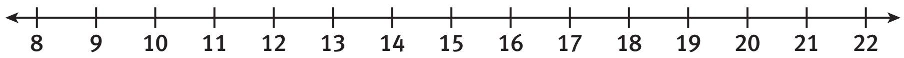


Not Fun

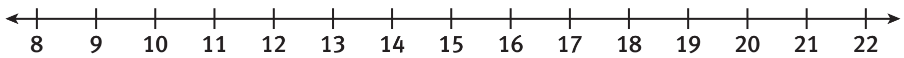
10. Using the data that Jamie and Riley collected, rearrange the "Perceived Minutes Elapsed" values in such a manner that they would **not** be statistically significant.

Group	Perceived Minutes Elapsed						
Fun							
Not Fun							

11. Create a dot plot for these groups, find the median for each, and compute the difference of the medians.



Fun



Not Fun

12. Explain why the difference in medians may be a good statistic to investigate to determine statistical significance.

Check Your Understanding

13. Find the difference in the mean of the two test groups. Would difference in means be a good statistic for determining statistical significance in this situation?

14. Find the difference in the standard deviations of the two test groups. Would difference in standard deviations be a good statistic for determining statistical significance in this situation?

My Notes

My Notes

LESSON 40-1 PRACTICE

Carol and Alina play soccer for a local college team. Based on anecdotal evidence, they think that there is a difference in a player's success rate of taking penalty kicks with their dominant foot compared to their non-dominant foot. They would like to test this hypothesis with an experiment. Carol arranges for each of the 11 starting players on her team to take ten penalty kicks with their dominant foot and ten penalty kicks with their non-dominant foot, and records the data.

15. What are the treatments in this experiment?

Alina collected the data in the table below.

	Number of Successful Penalty Kicks										
Dominant Foot	8	7	9	9	6	7	7	8	5	9	7
Non-Dominant Foot	6	7	8	9	5	8	7	6	5	8	8

16. Draw a dot plot for each distribution. Does it seem that Carol and Alina's hypothesis is supported?

17. To test their hypothesis with this data set, which test statistic would be better: difference in medians or difference in means?

18. Using the same data values, describe two distributions that would be more supportive of the hypothesis.

19. Describe the meaning of statistical significance in this context.

My Notes

Learning Target:

- Design and conduct a simulation to test statistical significance.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, KWL Chart, Role Play, Summarizing, Paraphrasing, Think Aloud, Debriefing

Using the data values from Jamie and Riley's original study, you grouped data values in Lesson 40-1 to create data sets with median differences that were statistically significant and that were not statistically significant. However, the question remains for the original data set collected by Jamie and Riley: Is the difference of medians, 7 minutes, statistically significant in their study?

To investigate this question, create a model to randomly select data values from the original data set. This will represent a situation in which the treatment had no effect on the perception of the passage of time. Recall that the data collected from the study included responses of 9, 10, 10, 10, 11, 12, 14, 14, 15, 15, 17, 17, 18, and 20 minutes.

1. Use the *randInt* function on your calculator to choose random integers from 9 to 20. Repeat the process until you obtain seven of the data values above, without replacement. (Note that 10, 14, 15, and 17 occur multiple times, and therefore can occur the same number of times in your selection.)

a. Write those seven data values as the "Fun" values. The values that remain are the "Not Fun" values. Use the table below to organize your selections.

Group	Perceived Minutes Elapsed						
Fun							
Not Fun							

b. Find the median of the "Fun" data values and the median of the "Not Fun" data values. Subtract the "Fun" median from the "Not Fun" median.

TECHNOLOGY TIP

To find the *randInt* function on the TI-84, press MATH and the arrow keys to select the *PRB* menu, and select *randInt(* . The first entry is the least integer from the range you would like to sample, followed by a comma, and then the greatest integer from the range. Press ENTER and the result is an integer, chosen at random, from the range you indicated.

For example, to choose a random integer between 5 and 15, including 5 and 15, enter *randInt(5,15)*.

My Notes

2. Repeat the process in Item 1 and record the difference of the medians in the table below.

Simulation Number	1	2	3	4	5	6	7	8	9	10
"Fun" Median – "Not Fun" Median										

3. Combine your list of ten differences with those of your classmates so that you have at least 100 values. (The more values you have, the better your results will be). Write the results in the table below.

Difference of Medians	Frequency
−7	
−6	
−5	
−4	
−3	
−2	
−1	
0	
1	
2	
3	
4	
5	
6	
7	

My Notes

4. Create a histogram with the combined class values from Item 3, and describe the shape of the distribution.

In Jamie and Riley's real data set, the difference between the median data values of the two groups was 7 minutes. That difference seemed rather large, but it wasn't obvious whether it was so large that it was statistically significant.

5. Describe the meaning of *statistically significant* in this context.

6. Based on your results, what is the probability of the difference in medians being as great as 7 minutes?

My Notes

ACADEMIC VOCABULARY

A **simulation** is a model of a real-world process in which imaginary data are generated, usually many times, to determine what results can be expected from the real-world process.

CONNECT TO AP

In AP Statistics, it is important to be able not only to draw an appropriate conclusion from data, but also to articulate a logical argument about how the conclusion follows from the data.

The ***simulation*** you just conducted is a tool to help determine what values are typical and what values are atypical, assuming that the subjects' responses would have been the same regardless of which treatment group they ended up in.

7. Based on the results of your simulation, is the 7-minute difference in medians statistically significant? Explain.

8. Write a logical argument explaining what Riley and Jamie should conclude from their experiment. Your argument should draw on the assumption behind the simulation, the result of the simulation, and Jamie and Riley's actual data. Review the draft of your argument. Be sure to check that you have described specific details, included the correct mathematical terms to support your reasoning, and that your sentences are complete and grammatically correct. You may want to pair-share with another student to critique each other's drafts and make improvements.

Check Your Understanding

9. In Jamie and Riley's experiment, are there other possible values that their test statistic might have taken which would have led them to the same conclusion—that "time flies when you are having fun"?

10. Write a conclusion that Jamie and Riley could have drawn if the difference between the medians in their study had been 3 minutes instead of 7 minutes.

My Notes

LESSON 40-2 PRACTICE

Use the following for Items 11–13.

In the experiment described in this activity, Jamie and Riley chose to use the difference between the median responses of subjects in the two groups as their test statistic. Suppose instead that they had decided to look at the ratio of the means from the two groups by dividing the mean perceived time in the "Not Fun" group by the mean perceived time in the "Fun" group.

11. Compute the mean perceived times for the "Not Fun" group and the "Fun" group, and then write the ratio.

12. Interpret this ratio in terms of the context of time perception between the "Not Fun" and "Fun" groups.

13. After completing many simulations, what would Jamie and Riley do next to test their hypothesis?

Use the following for Items 14–16.

Recall that in Jamie and Riley's study, they decided to interrupt the subjects' activity at 13.5 minutes. Suppose they had decided instead to interrupt them after 17 minutes.

14. What would have been different about the data?

15. How would the test statistics of difference in medians and ratio of means have changed?

16. Would Jamie and Riley's conclusions be different if the actual time that participants were involved with their activity were increased to 17 minutes? Explain your reasoning.

ACTIVITY 40 PRACTICE

**Write your answers on notebook paper.
Show your work.**

Lesson 40-1

Abraham and Luis are interested in conducting an experiment to see whether people's ability to successfully toss a toy ball into a basket is influenced by their belief that others found the task difficult or easy. They position themselves in a central location at their school, place a basket 15 feet away from a spot marked "X," and ask volunteers to try to make the basket. They randomly choose subjects to participate and randomly tell them one of two statements: "So far, only one-fourth of people have made it" or "So far, only one-fourth of people have missed it." They repeat this for a total of 100 subjects. Their data are summarized below.

	Number Who Made Shot	Number Who Missed Shot	Total
Told That Most People Made It	29	19	48
Told That Most People Missed It	25	27	52

1. What is the question of interest in this study?
2. What are the treatments imposed by Abraham and Luis in this experiment?
3. Abraham and Luis decided that their test statistic is the difference between the proportion of people who made the shot in the group that was told the task was easy and the proportion of the people who made the shot in the group that was told the task was difficult. Interpret the meaning of a positive test statistic, a negative test statistic, and a test statistic of zero in this context.
4. Compute the test statistic for the results of the experiment.
5. If it was determined that the test statistic was statistically significant, what would Abraham and Luis be able to conclude?
6. If it was determined that the test statistic was not statistically significant, what would Abraham and Luis be able to conclude?

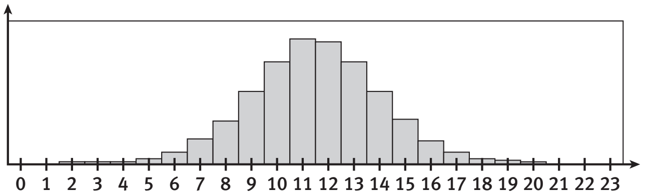
Lesson 40-2

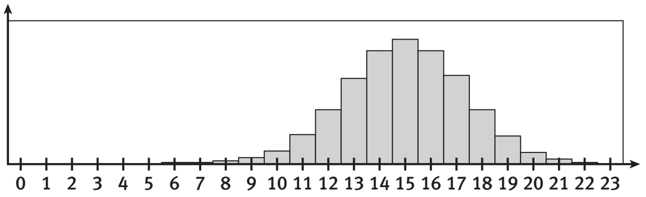
7. Abraham and Luis's teacher provided them with 200 beads, 100 red and 100 white, of which the only difference was their color. Describe how Abraham and Luis could use those beads to create a simulation to determine whether their test statistic is statistically significant. Be sure to identify what the beads represent.

Use the following for Items 8 and 9.

In a study designed to determine whether babies have an innate sense of morality, babies were shown two puppet shows in a random order: one of them had a puppet being nice, and the other had a different puppet being mean. The babies were then given the opportunity to reach for either the nice puppet or the mean puppet, and the researchers recorded which puppet the babies reached for. Suppose that out of 23 babies in the study, 15 of them reached for the nice puppet.

8. One of the distributions below shows the probability distribution of the number of babies who would reach for the nice puppet if, in fact, babies had no sense of morality and were reaching for a puppet at random. Which distribution is it, and how do you know?





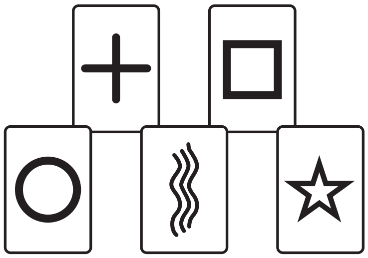
MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

9. Using the distribution you picked in Item 8 and the observed 15 out of 23 babies reaching for the nice puppet, what conclusion should be drawn, and why?

Simulations, Margin of Error, and Hypothesis Testing

PSYCHIC OR JUST HOT AIR?

1. "Zener cards" are used to test whether someone has extrasensory perception (ESP). Each card has one of five distinct images on it:



Suppose that a subject is presented with a random assortment of 12 such cards and is asked to identify the images without looking at them. He correctly identifies 6 out of the 12 cards.

a. Given a random card, what is the probability of correctly identifying the image on the card?

b. Use the random digits table below, with 0 and 1 representing correct identifications and digits 2-9 representing incorrect identifications, to perform ten different simulations. Beginning with row 113, record the number of successes for each trial, and make a dot plot of your results.

111	81486	69487	60513	09297	00412	71238	27649	39950
112	59636	88804	04634	71197	19352	73089	84898	45785
113	62568	70206	40325	03699	71080	22553	11486	11776
114	45149	32992	75730	66280	03819	56202	02938	70915
115	61041	77684	94322	24709	73698	14526	31893	32592

c. What conclusion does this data support?

2. An engineer developed a treatment that he hoped would make the fabric of a hot-air balloon last longer. Out of 9 volunteer balloonists with new balloons, he randomly selected 4 to get no special treatment, and 5 to get their fabric treated. The table below shows how many balloon-hours the nine balloons lasted.

No Special Treatment	520	610	435	443	
Received the Special Treatment	496	639	550	622	600

a. Find the difference in the means of the two groups.

b. Each balloon-hour total is written on an index card. Describe a simulation using these cards that could help determine the statistical significance of the difference of the means you found in part a.

c. Describe a manner in which the results of the simulation would allow you to reasonably conclude that the difference in the means was not statistically significant.

3. A regular survey asks a random sample of 1070 American adults whether they approve of the job the President of the United States is doing. The margin of error in the proportion of people who say "yes" is stated to be ± 3 percentage points.

a. Suppose such a survey yielded a proportion of 0.45. Explain what that means in everyday language.

b. How could the survey be conducted differently to reduce the margin of error?

Simulations, Margin of Error, and Hypothesis Testing

PSYCHIC OR JUST HOT AIR?

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3)	• Clear and accurate understanding of significance testing using a table of random digits or a simulation • Clear and accurate understanding of margin of error and survey design principles	• A functional understanding and accurate interpretation of significance testing using a table of random digits or a simulation • A functional and mostly accurate understanding of margin of error and survey design principles	• Partial understanding and partially accurate interpretation of significance testing using a table of random digits or a simulation • Partial understanding and partially accurate work with margin of error and survey design principles	• Little or no understanding and inaccurate interpretation of significance testing using a table of random digits or a simulation • Little or no understanding and inaccurate work with margin of error and survey design principles
Problem Solving (Items 1, 2, 3)	• An appropriate and efficient strategy that results in a correct answer	• A strategy that may include unnecessary steps but results in a correct answer	• A strategy that results in some incorrect answers	• No clear strategy when solving problems
Mathematical Modeling / Representations (Items 1, 2)	• Clear and accurate understanding of how to apply simulations and random digit tables to analyze real-world scenarios	• Mostly accurate understanding of how to apply simulations and random digit tables to analyze real-world scenarios	• Partial understanding of how to apply simulations and random digit tables to analyze real-world scenarios	• Inaccurate or incomplete understanding of how to apply simulations and random digit tables to analyze real-world scenarios
Reasoning and Communication (Items 1, 2, 3)	• Precise use of appropriate math terms and language to describe margin of error and how to reduce it in a survey • Clear and accurate explanation of methods to determine statistical significance	• Adequate description of margin of error and how to reduce it in a survey • Adequate explanation of methods to determine statistical significance	• Misleading or confusing description of margin of error and how to reduce it in a survey • Misleading or confusing explanation of methods to determine statistical significance	• Incomplete or inaccurate description of margin of error and how to reduce it in a survey • Incomplete or inadequate explanation of methods to determine statistical significance