

Equations, Inequalities, Functions

1

Unit Overview

In this unit, you will model real-world situations by using one- and two-variable linear equations. You will extend your knowledge of linear relationships through the study of inverse functions, composite functions, piecewise-defined functions, operations on functions, and systems of linear equations and inequalities.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- interpret
- compare
- contrast
- feasible
- confirm
- prove

Math Terms

- absolute value equation
- absolute value inequality
- constraints
- consistent
- inconsistent
- independent
- dependent
- ordered triple
- Gaussian elimination
- matrix
- dimensions of a matrix
- square matrix
- multiplicative identity matrix
- multiplicative inverse matrix
- matrix equation
- coefficient matrix
- variable matrix
- constant matrix
- piecewise-defined function
- step function
- parent function
- composition
- composite function
- inverse function

ESSENTIAL QUESTIONS



How are linear equations and systems of equations and inequalities used to model and solve real-world problems?



How are composite and inverse functions useful in problem solving?

EMBEDDED ASSESSMENTS

This unit has two embedded assessments, following Activities 3 and 6. They will give you an opportunity to demonstrate your understanding of equations, inequalities, and functions.

Embedded Assessment 1:

Equations, Inequalities, and Systems p. 55

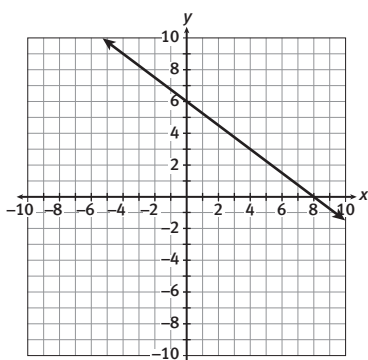
Embedded Assessment 2:

Piecewise-Defined, Composite, and Inverse Functions p. 99

Getting Ready

Write your answers on notebook paper.
Show your work.

- Given $f(x) = x^2 - 4x + 5$, find each value.
 - $f(2)$
 - $f(-6)$
- Find the slope and y -intercept.
 - $y = 3x - 4$
 - $4x - 5y = 15$
- Graph each equation.
 - $2x + 3y = 12$
 - $x = 7$
- Write an equation for each line.
 - line with slope 3 and y -intercept -2
 - line passing through $(2, 5)$ and $(-4, 1)$
- Write the equation of the line below.



- Using the whole number 5, define the additive inverse and the multiplicative inverse.
- Solve $3(x + 2) + 4 = 5x + 7$.
- What is the absolute value of 2 and of -2 ? Explain your response.
- Solve the equation for x .

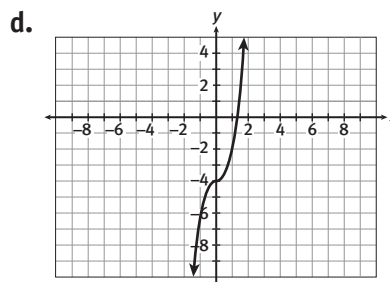
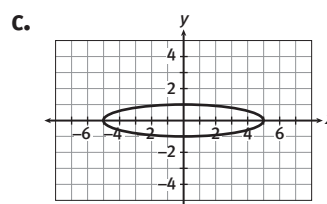
$$\frac{3x + y}{z} = 2$$

- Which point is a solution to the equation $6x - 5y = 4$? Justify your choice.
 - $(1, 2)$
 - $(1, -2)$
 - $(-1, -2)$
 - $(-1, 2)$
- Find the domain and range of each relation.

a. $y = 2x + 1$

b.

input	3	7	11
output	-1	-3	-5



- How many lines of symmetry exist in the figure shown in Item 11c?

Creating Equations

One to Two

Lesson 1-1 One-Variable Equations

Learning Targets:

- Create an equation in one variable from a real-world context.
- Solve an equation in one variable.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Activating Prior Knowledge, Create Representations, Identify a Subtask, Think-Pair-Share, Close Reading

A new water park called Sapphire Island is about to have its official grand opening. The staff is putting up signs to provide information to customers before the park opens to the general public. As you read the following scenario, mark the text to identify key information and parts of sentences that help you make meaning from the text.

The Penguin, one of the park's tube rides, has two water slides that share a single line of riders. The table presents information about the number of riders and tubes that can use each slide.

Penguin Water Slides

Slide Number	Tube Size	Tube Release Time
1	2 riders	every 0.75 min
2	4 riders	every 1.25 min

Jaabir places a sign in the waiting line for the Penguin. When a rider reaches the sign, there will be approximately 100 people in front of him or her waiting for either slide. The sign states, "From this point, your wait time is approximately ____ minutes." Jaabir needs to determine the number of minutes to write on the sign. Work with a partner or with your group on Items 1–7.

1. Let the variable r represent the number of riders taking slide 1. Write an *algebraic expression* for the number of tubes this many riders will need, assuming each tube is full.
2. Next, write an expression for the time in minutes it will take r riders to go down slide 1.
3. Assuming that r riders take slide 1 and that there are 100 riders in all, write an expression for the number of riders who will take slide 2.

My Notes

MATH TIP

An *algebraic expression* includes at least one variable. It may also include numbers and operations, such as addition, subtraction, multiplication, and division. It does not include an equal sign.

My Notes

- Using the expression you wrote in Item 3, write an expression for the number of tubes the riders taking slide 2 will need, assuming each tube is full.
- Write an expression for the time in minutes needed for the riders taking slide 2 to go down the slide.
- Since Jaabir wants to know how long it takes for 100 riders to complete the ride when both slides are in use, the total time for the riders taking slide 1 should equal the total time for the riders taking slide 2. Write an equation that sets your expression from Item 2 (the time for the slide 1 riders) equal to your expression from Item 5 (the time for the slide 2 riders).
- Reason abstractly and quantitatively.** Solve your equation from Item 6. Describe each step to justify your solution.

MATH TIP

These properties of real numbers can help you solve equations.

Addition Property of Equality

If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality

If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality

If $a = b$, then $ca = cb$.

Division Property of Equality

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Distributive Property

$a(b + c) = ab + ac$

Lesson 1-1

One-Variable Equations

ACTIVITY 1

continued

- 8. Make sense of problems.** Consider the meaning of the solution from Item 7.
- Explain why you should or should not round the value of r to the nearest whole number.
 - How many people out of the 100 riders will take slide 1?
- 9.** Use the expression you wrote in Item 2 to determine how long it will take the number of riders from Item 8b to go through slide 1.
- Evaluate* the expression for the appropriate value of r .
 - How many minutes will it take the riders to go through slide 1? Round to the nearest minute.

The rest of the 100 riders will go through slide 2 in about the same amount of time. So, your answer to Item 9b gives an estimate of the number of minutes it will take all 100 riders to go down the Penguin slides.

- 10.** Recall that when a rider reaches the sign, there will be approximately 100 people waiting in front of him or her. What number should Jaabir write to complete the statement on the sign?

From this point, your wait is approximately _____ minutes.

My Notes

MATH TIP

When you *evaluate* an algebraic expression, you substitute values for the variables and then simplify the expression.

My Notes

11. Describe how you could check that your answer to Item 10 is reasonable.

Check Your Understanding

12. Suppose that Jaabir needs to place a second sign in the waiting line for the Penguin slides. When a rider reaches this sign, there will be approximately 250 people in front of him or her. What number should Jaabir write to complete the statement on this sign? Explain how you determined your answer.

From this point, your wait is approximately _____ minutes.

13. Explain the relationships among the terms *variable*, *expression*, and *equation*.

LESSON 1-1 PRACTICE

Use this information for Items 14–15. When full, one of the pools at Sapphire Island will hold 43,000 gallons of water. The pool currently holds 20,000 gallons of water and is being filled at a rate of 130 gallons per minute.

14. Write an equation that can be used to find h , the number of hours it will take to fill the pool from its current level. Explain the steps you used to write your equation.
15. Solve your equation from Item 14, and *interpret* the solution.

Use this information for Items 16–18. Sapphire Island is open 7 days a week. The park has 8 ticket booths, and each booth has a ticket seller from 10:00 a.m. to 5 p.m. On average, ticket sellers work 30 hours per week.

16. **Model with mathematics.** Write an equation that can be used to find t , the minimum number of ticket sellers the park needs. Explain the steps you used to write your equation.
17. Solve your equation from Item 16, and interpret the solution.
18. The park plans to hire 20 percent more than the minimum number of ticket sellers needed in order to account for sickness, vacation, and lunch breaks. How many ticket sellers should the park hire? Explain.

ACADEMIC VOCABULARY

When you *interpret* a solution, you state the meaning of the solution in the context of the problem or real-world situation.

My Notes

4. What does the dependent variable y represent in this situation? Explain.

5. Write an equation that models the situation.

6. How can you tell whether the equation you wrote in Item 5 correctly models the situation?

7. **Construct viable arguments.** Explain how an employee could use the equation to determine how much to charge a customer.

Maria also thinks it would be useful to make a graph of the equation that relates the time in hours a tube is rented and the cost in dollars of renting a tube.

8. List five ordered pairs that lie on the graph of the relationship between x and y .

MATH TIP

Before you can graph the equation, you need to determine the coordinates of several points that lie on its graph. One way to do this is by using pairs of corresponding values from the table on the previous page. You can also choose several values of x and substitute them into the equation to determine the corresponding values of y .

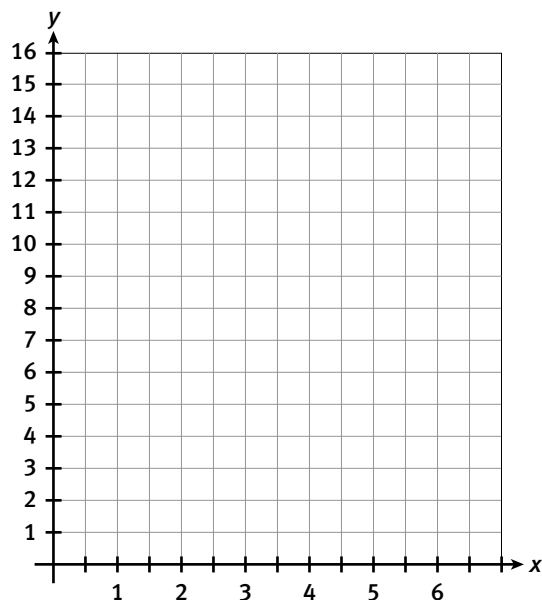
Lesson 1-2

Two-Variable Equations

ACTIVITY 1

continued

9. Use the grid below to complete parts a and b.
 - a. Write an appropriate title for the graph based on the real-world situation. Also write appropriate titles for the x - and y -axes.
 - b. Graph the ordered pairs you listed in Item 8. Then connect the points with a line or a smooth curve.



10. Based on the graph, explain how you know whether the equation that models this situation is or is not a *linear equation*.
11. **Reason quantitatively.** Explain why the graph is only the first quadrant.
12. What is the *y-intercept* of the graph? Describe what the *y-intercept* represents in this situation.
13. What is the slope of the graph? Describe what the slope represents in this situation.

My Notes

MATH TIP

Recall that a *linear equation* is an equation whose graph is a line. A linear equation can be written in *standard form* $Ax + By = C$, where A , B , and C are integers and A is nonnegative.

MATH TIP

The *y-intercept* of a graph is the y -coordinate of a point where the graph intersects the y -axis.

The *slope* of a line is the ratio of the change in y to the change in x between any two points.

My Notes

DISCUSSION GROUP TIP

Share your description with your group members and list any details you may not have considered before. If you do not know the exact words to describe your ideas, use synonyms or request assistance from group members to help you convey your ideas. Use nonverbal cues such as raising your hand to ask for clarification of others' ideas.

14. Work with your group. Describe a plausible scenario related to the water park that could be modeled by this equation: $y = 40x - 8$. In your description, be sure to use appropriate vocabulary, both real-world and mathematical. Refer to the Word Wall and any notes you may have made to help you choose words for your description.

Check Your Understanding

15. Explain why the slope of the line you graphed in Item 9 is positive.
16. Explain how you would graph the equation from Item 14. What quantity and units would be represented on each axis?
17. Is the equation $y = -2x + x^2$ a linear equation? Explain how you know.

LESSON 1-2 PRACTICE

Use this information for Items 18–22. Some of the water features at Sapphire Island are periodically treated with a chemical that prevents algae growth. The directions for the chemical say to add 16 fluid ounces per 10,000 gallons of water.

18. Make a table that shows how much of the chemical to add for water features that hold 10,000; 20,000; 30,000; 40,000; and 50,000 gallons of water.
19. Write a linear equation in two variables that models the situation. Tell what each variable in the equation represents.
20. Graph the equation. Be sure to include titles and use an appropriate scale on each axis.
21. What are the slope and y -intercept of the graph? What do they represent in the situation?
22. **Construct viable arguments.** An employee adds 160 fluid ounces of the chemical to a feature that holds 120,000 gallons of water. Did the employee add the correct amount? Explain.

My Notes

MATH TIP

You know that the distance from t to 82°F on a thermometer is 4.5°F . This distance can be modeled with the absolute value expression $|t - 82|$.

- 1. Reason abstractly.** How many solutions are possible for an absolute value equation having the form $|ax + b| = c$, where a , b , and c are real numbers?

Example B

The temperature of the wave pool at Sapphire Island can vary up to 4.5°F from the target temperature of 82°F . Write and solve an absolute value equation to find the temperature extremes of the wave pool. (The temperature extremes are the least and greatest possible temperatures.)

Step 1: Write an absolute value equation to represent the situation.

Let t represent the temperature extremes of the wave pool in degrees Fahrenheit.

$$|t - 82| = 4.5$$

Step 2: Use the definition of absolute value to solve for t .

$$|t - 82| = 4.5$$

$$t - 82 = 4.5 \quad \text{or} \quad t - 82 = -4.5$$

$$t = 86.5 \quad \text{or} \quad t = 77.5$$

Solution: The greatest possible temperature of the wave pool is 86.5°F , and the least possible temperature is 77.5°F . Both of these temperatures are 4.5°F from the target temperature of 82°F .

Try These B

The pH of water is a measure of its acidity. The pH of the water on the Seal Slide can vary up to 0.3 from the target pH of 7.5. Use this information for parts a–c.

- Write an absolute value equation that can be used to find the extreme pH values of the water on the Seal Slide. Be sure to explain what the variable represents.
- Solve your equation, and interpret the solutions.
- Reason quantitatively.** Justify the reasonableness of your answer to part b.

Lesson 1-3

Absolute Value Equations and Inequalities

ACTIVITY 1

continued

My Notes

Solving **absolute value inequalities** algebraically is similar to solving absolute value equations. By the definition of absolute value, $|ax + b| > c$, where $c > 0$, is equivalent to $-(ax + b) > c$ or $ax + b > c$. Multiplying the first inequality by -1 , and then using a similar method for $|ax + b| < c$, gives these statements:

- $|ax + b| > c$, $c > 0$, is equivalent to $ax + b < -c$ or $ax + b > c$.
- $|ax + b| < c$, $c > 0$, is equivalent to $ax + b < c$ or $ax + b > -c$, which can also be written as $-c < ax + b < c$.

MATH TERMS

An **absolute value inequality** is an inequality involving the absolute value of a variable expression.

Example C

Solve each inequality. Graph the solutions on a number line.

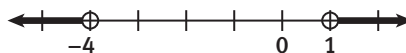
a. $|2x + 3| + 1 > 6$

Step 1: Isolate the absolute value expression. $|2x + 3| + 1 > 6$
 $|2x + 3| > 5$

Step 2: Write two inequalities. $2x + 3 > 5$ or $2x + 3 < -5$

Step 3: Solve each inequality. $x > 1$ or $x < -4$

Solution:



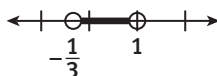
b. $|3x - 1| + 5 < 7$

Step 1: Isolate the absolute value expression. $|3x - 1| + 5 < 7$
 $|3x - 1| < 2$

Step 2: Write the compound inequality. $-2 < 3x - 1 < 2$

Step 3: Solve the inequality. $-\frac{1}{3} < x < 1$

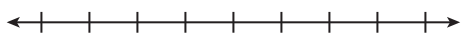
Solution:



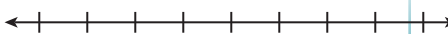
Try These C

Solve and graph each absolute value inequality.

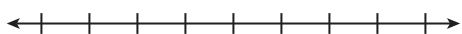
a. $|x - 2| > 3$



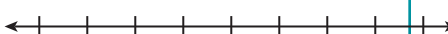
b. $|x + 2| - 3 \leq -1$



c. $|5x - 2| + 1 \geq 4$



d. $|2x + 7| - 4 < 1$



MATH TIP

These properties of real numbers can help you solve inequalities. The properties also apply to inequalities that include $<$, \geq , or \leq .

Addition Property of Inequality

If $a > b$, then $a + c > b + c$.

Subtraction Property of Inequality

If $a > b$, then $a - c > b - c$.

Multiplication Property of Inequality

If $a > b$ and $c > 0$, then $ca > cb$.

If $a > b$ and $c < 0$, then $ca < cb$.

Division Property of Inequality

If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

ACTIVITY 1 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 1-1

Susan makes and sells purses. The purses cost her \$12 each to make, and she sells them for \$25. This Saturday, she is renting a booth at a craft fair for \$60. Use this information for Items 1–3.

- Write an equation that can be used to find the number of purses Susan must sell to make a profit of \$250 at the fair.
- Solve the equation, and interpret the solution.
- If Susan sells 20 purses at the fair, will she meet her profit goal? Explain why or why not.

A medical rescue helicopter is flying at an average speed of 172 miles per hour toward its base hospital. At 2:42 p.m., the helicopter is 80 miles from the hospital. Use this information for Items 4–6.

- Which equation can be used to determine m , the number of minutes it will take the helicopter to reach the hospital?

A. $172(60m) = 80$ B. $172\left(\frac{m}{60}\right) = 80$

C. $172\left(\frac{60}{m}\right) = 80$ D. $\frac{172}{60m} = 80$
- Solve the equation, and interpret the solution.
- An emergency team needs to be on the roof of the hospital 3 minutes before the helicopter arrives. It takes the team 4 minutes to reach the roof. At what time should the team start moving to the roof to meet the helicopter? Explain your reasoning.

Jerome bought a sweater that was on sale for 20 percent off. Jerome paid \$25.10 for the sweater, including sales tax of 8.25 percent. Use this information for Items 7–9.

- Write an equation that can be used to find the original price of the sweater.
- Solve the equation, and interpret the solution.
- How much money did Jerome save by buying the sweater on sale? Explain how you determined your answer.

Lesson 1-2

A taxi company charges an initial fee of \$3.50 plus \$2.00 per mile. Use this information for Items 10–16.

- Make a table that shows what it would cost to take a taxi for trips of 1, 2, 3, 4, and 5 miles.
- Write an equation in two variables that models this situation. Explain what the independent variable and the dependent variable represent.
- Graph the equation. Be sure to include a title for the graph and for each axis.
- Describe one advantage of the graph compared to the equation.
- Is the equation that models this situation a linear equation? Explain why or why not.
- What are the slope and y -intercept of the graph? What do they represent in the situation?
- Shelley uses her phone to determine that the distance from her apartment to Blue Café is 3.7 miles. How much will it cost Shelley to take a taxi to the café?

ACTIVITY 1

continued

17. Choose the equation that is *not* linear.

- A. $y = \frac{2}{3}x + 6$ B. $y = \frac{4}{x} - 1$
 C. $3x + 2y = 8$ D. $x = -4$

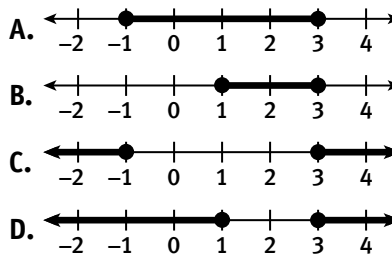
A zoo is building a new large-cat exhibit. Part of the space will be used for lions and part for leopards. The exhibit will house eight large cats in all. Expenses for a lion will be about \$8000 per year, and expenses for a leopard will be about \$6000 per year. Use this information for Items 18–21.

18. Write an equation that can be used to find y , the yearly expenses for the eight cats in the exhibit when x of the cats are lions.
 19. Graph the equation. Be sure to include a title for the graph and for each axis.
 20. Are all points on the line you graphed solutions in this situation? Explain.
 21. What would the yearly expenses be if five of the cats in the exhibit are lions and the rest are leopards? Explain how you found your answer.

Lesson 1-3

22. Solve each absolute value equation.
 a. $|2x - 3| = 7$
 b. $|2x + 5| = 23$
 c. $|x - 10| - 11 = 12 - 23$
 d. $|7x + 1| - 7 = 3$
 e. $|2x| - 3 = -5$
23. If the center thickness of a lens varies more than 0.150 millimeter from the target thickness of 5.000 millimeters, the lens cannot be used. Write and solve an absolute value equation to find the extreme acceptable values for the center thickness of the lens.
24. Solve the equation $|2x + 4| - 1 = 7$. Then graph the solutions on a number line.

25. A thermometer is accurate to within 0.6°F . The thermometer indicates that Zachary's temperature is 101.7°F . Write and solve an absolute value equation to find the extreme possible values of Zachary's actual temperature.
26. Solve each absolute value inequality. Graph the solutions on a number line.
 a. $|x + 5| < 12$
 b. $|5x + 2| \geq 13$
 c. $|10x - 12| - 9 \leq -1$
 d. $|x - 7| + 3 > 8$
 e. $|-2x + 5| + 6 \geq 4$
27. Which number line shows the solutions of the inequality $2|x - 1| \geq 4$?



MATHEMATICAL PRACTICES

Attend to Precision

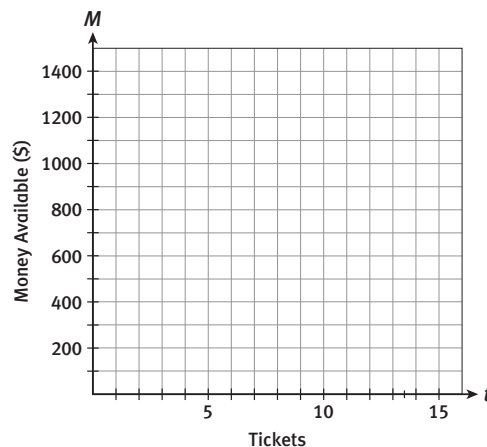
28. The equation $y = 0.5x + 40$ represents the monthly cost y in dollars of Lesley's cell phone, where x is the number of talk minutes over 750 that Lesley uses.
 a. Graph the equation.
 b. How did you determine the range of values to show on each axis of your graph?
 c. What are the units on each axis of your graph?
 d. What are the units of the slope of the linear equation? Explain.
 e. Write a different plausible scenario—not related to cell phone costs—that could be modeled using the equation $y = 0.5x + 40$. Be sure to use appropriate vocabulary, both real-world and mathematical.

© 2015 College Board. All rights reserved.

My Notes

2. **Model with mathematics.** Roy wants to know how the purchase of each ticket affects his available money. Fill in the table below. Plot the points on the grid.

Tickets (t)	Money Available (M)
0	
1	
2	
3	
4	
5	
8	
10	
13	



3. What patterns do you notice?
4. Explain how you determined the values for 8, 10, and 13 tickets.
5. Write a *function* $M(t)$ that represents the amount of money that Roy has left after purchasing t tickets.

MATH TIP

A *function* is a relationship between two quantities in which each input has exactly one output.

Lesson 2-1

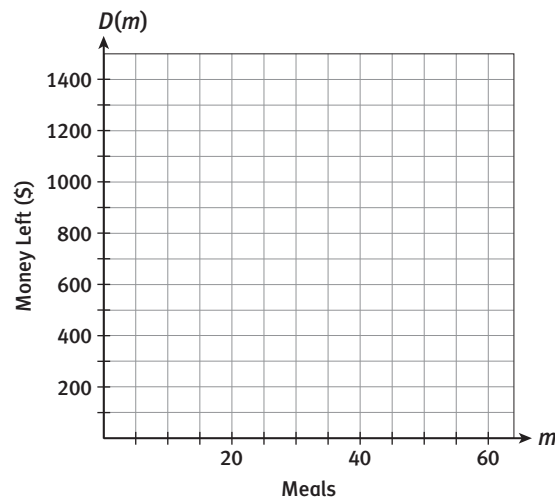
Graphing Two-Variable Equations

ACTIVITY 2

continued

- Use mathematical terminology to explain what -100 and 1360 each represent in your function in Item 5.
- Roy wonders how his meal costs will affect his spending money.
 - Write a function $D(m)$ that represents the amount of money Roy has left after purchasing m number of meals.

- Graph your function on the grid.



- What kind of function is $D(m)$?
- What is the *rate of change* for $D(m)$, including units?
- Make sense of problems.** Are all the values for m on your graph valid in this situation, given that m represents the number of meals that Roy can buy? Explain.

My Notes

CONNECT TO TECHNOLOGY

You can also graph the function by using a graphing calculator. When entering the equation, use x for the independent variable and y for the dependent variable.

MATH TIP

The *rate of change* of a function is the ratio of the amount of change in the dependent variable to the amount of change in the independent variable.

My Notes

MATH TIP

The *slope-intercept form* of a line is $y = mx + b$, where m is the slope and b is the y -intercept.

Check Your Understanding

11. What do the x - and y -intercepts of your graph in Item 7 represent?
12. If you know the coordinates of two points on the graph of a linear function, how can you determine the function's rate of change?
13. What is the relationship between the rate of change of a linear function and the slope of its graph?
14. Using your answers to Items 12 and 13, explain how to write the equation of a line when you are given the coordinates of two points on the line.

LESSON 2-1 PRACTICE

15. Write the equation of the line with y -intercept -4 and a slope of $\frac{3}{2}$. Graph the equation.
16. Write the equation of the line that passes through the point $(-2, -3)$ and has a slope of 5 . Graph the equation.
17. **Model with mathematics.** Graph the function $f(x) = 3 - \frac{1}{2}(x - 2)$.

Use the following information for Items 18–20. Roy already has 10,368 frequent flyer miles, and he will earn 2832 more miles from his round-trip flight to New York City. In addition, he earns 2 frequent flyer miles for each dollar he charges on his credit card.

18. Write the equation of a function $f(d)$ that represents the total number of frequent flyer miles Roy will have after his trip if he charges d dollars on his credit card.
19. Graph the function, using appropriate scales on the axes.
20. **Reason quantitatively.** How many dollars will Roy need to charge on his credit card to have a total of 15,000 frequent flyer miles? Explain how you determined your answer.

Learning Targets:

- Represent constraints by equations or inequalities.
- Use a graph to determine solutions of a system of inequalities.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Interactive Word Wall, Create Representations, Work Backward, Discussion Groups, Close Reading, Debriefing, Activating Prior Knowledge

Work with your group on Items 1 through 5. As needed, refer to the Glossary to review translations of key terms. Incorporate your understanding into group discussions to confirm your knowledge and use of key mathematical language.

1. Roy's spending money depends on both the number of tickets t and the number of meals m . Determine whether each option is *feasible* for Roy and provide a rationale in the table below.

Tickets (t)	Meals (m)	Total Cost	Is it feasible?	Rationale
6	16			
8	14			
10	12			
4.5	11			

2. **Construct viable arguments.** For all the ordered pairs (t, m) that are feasible options, explain why each statement below must be true.
 - a. All coordinates in the ordered pairs are integer values.
 - b. If graphed in the coordinate plane, all ordered pairs would fall either in the first quadrant or on the positive m -axis.
3. Write a **linear inequality** that represents all ordered pairs (t, m) that are feasible options for Roy.
4. If Roy buys exactly two meals each day, determine the total number of tickets that he could purchase in five days. Show your work.

My Notes

ACADEMIC VOCABULARY

The term **feasible** means that something is possible in a given situation.

DISCUSSION GROUP TIPS

As you share your ideas, be sure to use mathematical terms and academic vocabulary precisely. Make notes as you listen to group members to help you remember the meaning of new words and how they are used to describe mathematical concepts. Ask and answer questions clearly to aid comprehension and to ensure understanding of all group members' ideas.

MATH TERMS

A **linear inequality** is an inequality that can be written in one of these forms, where A and B are not both equal to 0:
 $Ax + By < C$, $Ax + By > C$,
 $Ax + By \leq C$, or $Ax + By \geq C$.

My Notes

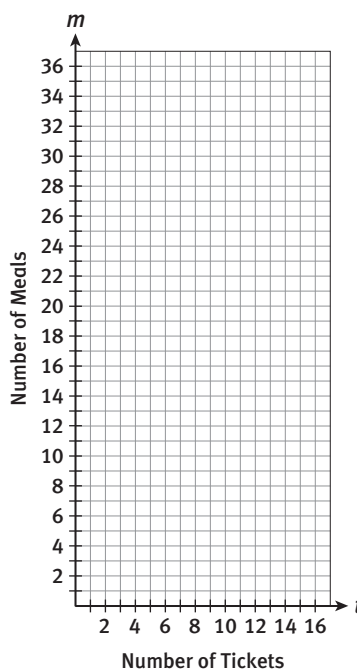
MATH TIP

Recall how to graph linear inequalities. First, graph the corresponding linear equation. Then choose a test point not on the line to determine which half-plane contains the set of solutions to the inequality. Finally, shade the half-plane that contains the solution set.

5. If Roy buys exactly one ticket each day, find the maximum number of meals that he could eat in the five days. Show your work.

6. To see what the feasible options are, you can use a visual display of the values on a graph.

a. **Attend to precision.** Graph your inequality from Item 3 on the grid below.



b. What is the boundary line of the graph?

c. Which half-plane is shaded? How did you decide?

My Notes

MATH TIP

Use a solid boundary line for inequalities that include \geq or \leq .
Use a dashed boundary line for inequalities that include $>$ or $<$.

MATH TERMS

Constraints are the conditions or inequalities that limit a situation.

Check Your Understanding

8. Compare and contrast the two graphs of the linear inequality: the one you made using paper and pencil and the one on your graphing calculator. Describe an advantage of each graph compared to the other.
 9.
 - a. What part of your graphs represents solutions for which Roy would have *no* money left over? Explain.
 - b. What part of your graphs represents solutions for which Roy would have money left over? Explain.
 10. Explain how you would graph the inequality $2x + 3y < 12$, either by using paper and pencil or by using a graphing calculator.
-
11. Roy realized that some other conditions or **constraints** apply. Write an inequality for each constraint described below.
 - a. Roy eats lunch and dinner the first day. On the remaining four days, Roy eats at least one meal each day, but he never eats more than three meals each day.
 - b. There are only 10 performances playing that Roy actually wants to see while he is in New York City, but he may not be able to attend all of them.
 - c. Roy wants the number of meals that he eats to be no more than twice the number of performances that he attends.
 12. **Model with mathematics.** You can use a graph to organize all the constraints on Roy's trip to New York City.
 - a. List the inequalities you found in Items 3 and 11.
 - b. Graph the inequalities from Items 3 and 11 on a single grid.

Lesson 2-2

Graphing Systems of Inequalities

ACTIVITY 2

continued

13. By looking at your graph, identify two ordered pairs that are feasible options to all of the inequalities. **Confirm** that these ordered pairs satisfy the inequalities listed in Item 12.
- First ordered pair (t, m) :
 - Second ordered pair (t, m) :
14. Label the point $(6, 10)$ on the grid in Item 6.
- Interpret the meaning of this point.
 - Construct viable arguments.** Is this ordered pair in the solution region common to all of the inequalities? Explain.
15. If Roy uses his prize money to purchase 6 tickets and eat 10 meals, how much money will he have left over for other expenses? Show your work.

Check Your Understanding

16. Given the set of constraints described earlier, how many tickets could Roy purchase if he buys 12 meals? Explain.
17. **a.** If you were Roy, how many meals and how many tickets would you buy during the 5-day trip?
b. Explain why you made the choices you did, and tell how you know that this combination of meals and tickets is feasible.
18. Explain how you would graph this constraint on a coordinate plane:
 $2 \leq x \leq 5$.

My Notes

ACADEMIC VOCABULARY

When you **confirm** a statement, you show that it is true or correct.

My Notes

LESSON 2-2 PRACTICE

19. Graph these inequalities on the same grid, and shade the solution region that is common to all of the inequalities: $y \geq 2$, $x \leq 8$, and $y \leq 2 + \frac{1}{2}x$.
20. Identify two ordered pairs that satisfy the constraints in Item 19 and two ordered pairs that do not satisfy the constraints.

A snack company plans to package a mixture of almonds and peanuts. The table shows information about these types of nuts. The company wants the nuts in each package to have at least 60 grams of protein and to cost no more than \$4. Use this information for Items 21–23.

Nut	Protein (g/oz)	Cost (\$/oz)
Almonds	6	0.30
Peanuts	8	0.20

MATH TIP

When answering Item 21, remember that the number of ounces of each type of nut cannot be negative.

21. **Model with mathematics.** Write inequalities that model the constraints in this situation. Let x represent the number of ounces of almonds in each package and y represent the number of ounces of peanuts.
22. Graph the constraints. Shade the solution region that is common to all of the inequalities.
23. a. Identify two ordered pairs that satisfy the constraints.
 b. **Reason quantitatively.** Which ordered pair represents the more expensive mixture? Which ordered pair represents the mixture with more protein? Explain your answer.

ACTIVITY 2 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 2-1

- Write the equation of the line with y -intercept 3 and a slope of $-\frac{2}{3}$, and graph it.
- Write the equation of the line that passes through the point $(-2, 4)$ and has a slope of -1 . Then graph it.
- Write the equation of the line in standard form that passes through the points $(2, -3)$ and $(-1, -4)$.

A jeweler is heating a gold bar. It takes 7 joules of heat to raise the temperature of the bar 1°C . The initial temperature of the bar is 25°C . Use this information for Items 4–11.

- Make a table that shows how many joules of heat would be required to raise the temperature of the gold bar to 26°C , 27°C , 28°C , 29°C , 30°C , and 35°C .
 - Write the equation of a function $h(t)$ that represents the amount of heat in joules required to heat the bar to a temperature of t degrees Celsius.
 - Graph the function. Be sure to label the axes.
 - What is the rate of change of the function, including units?
 - What is the t -intercept of the graph of the function? What does it represent in this situation?
 - Explain what the ordered pair $(32, 49)$ represents in this situation.
 - How many joules of heat will be required to heat the gold bar to a temperature of 260°C ? Explain how you determined your answer.
 - Explain what a negative value of $h(t)$ represents in this situation.
- A college tennis coach needs to purchase tennis balls for the team. A case of 24 cans costs \$60, and each can holds 3 balls. Use this information for Items 12–17.
- Write the equation of a function $c(t)$ that represents the number of cases the coach will need to purchase to have a total of t tennis balls.
 - Graph the function. Be sure to label the axes.
 - What is the slope of the graph?
 - $\frac{1}{72}$
 - $\frac{1}{8}$
 - 8
 - 72
 - What does the slope represent in this situation?
 - Are all the values for t on your graph valid in this situation, given that the coach can only buy complete cases of tennis balls? Explain.
 - The coach needs to purchase 600 tennis balls.
 - How many cases will the coach need to purchase to have this number of tennis balls? Explain how you determined your answer.
 - What is the actual number of tennis balls the coach will have when he buys this number of cases?
 - The coach has a budget of \$500 to buy tennis balls. Is there enough money to buy the number of cases that the coach needs? Explain.
 - Without graphing the equations, explain how you can tell which one represents the steeper line: $y = 5 + 5(x + 4)$ or $y = 2(3x - 2)$.

Lesson 2-2

19. Graph the inequality $y > 2x - 5$.
20. a. Graph the inequality $6x - 2y \geq 12$.
 b. Did you use a solid or dashed line for the boundary line? Explain your choice.
 c. Did you shade above or below the boundary line? Explain your choice.
21. Graph the following inequalities on the same grid and shade the solution region that is common to all of the inequalities.
- a. $y \geq 6 - \frac{2}{3}x$
 b. $y \leq 4 + x$
 c. $y \leq 11 - 8(x - 7)$

Catelyn has two summer jobs. Each week, she works at least 15 hours at a pet store and at least 6 hours as a nanny. She earns \$10 per hour at the pet store and \$8 per hour as a nanny. Catelyn wants to work no more than 30 hours and earn at least \$250 per week. Use this information for Items 22–25.

22. Let x represent the number of hours Catelyn works at the pet store in one week and y represent the number of hours she works as a nanny in one week. Write inequalities that model the four constraints in this situation.
23. Graph the constraints. Shade the solution region that is common to all of the inequalities.
24. a. Identify two ordered pairs that satisfy the constraints.
 b. Which ordered pair represents Catelyn working a greater number of hours? Which ordered pair represents Catelyn earning more money? Explain your answer.
25. a. Identify two ordered pairs that do not satisfy the constraints.
 b. For each ordered pair, identify the constraint or constraints that it fails to meet.

A tent designer is working on a new tent. The tent will be made from black fabric, which costs \$6 per yard, and green fabric, which costs \$4 per yard. The designer will need at least 3 yards of black fabric, at least 4 yards of green fabric, and at least 10 yards of fabric overall. The total cost of the fabric used for the tent can be no more than \$60. Use this information for Items 26–28.

26. Let x represent the number of yards of black fabric and y represent the number of yards of green fabric. Write inequalities that model the four constraints in this situation.
27. Graph the constraints. Shade the solution region that is common to all of the inequalities.
28. Which ordered pair lies in the solution region that is common to all of the inequalities?
 A. (2, 12) B. (4, 7)
 C. (6, 8) D. (10, 3)

MATHEMATICAL PRACTICES**Reason Abstractly and Quantitatively**

Look back at the scenario involving the tent designer.

29. a. What does the ordered pair you chose in Item 28 represent in the situation?
 b. What is the greatest amount of green fabric the designer can use if all of the constraints are met? Explain your answer.
 c. What is the least amount of black fabric the designer can use if all of the constraints are met? Explain.

Systems of Linear Equations

Monetary Systems Overload

Lesson 3-1 Solving Systems of Two Equations in Two Variables

Learning Targets:

- Use graphing, substitution, and elimination to solve systems of linear equations in two variables.
- Formulate systems of linear equations in two variables to model real-world situations.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Close Reading, Create Representations, Discussion Groups, Role Play, Think-Pair-Share, Quickwrite, Note Taking, Look for a Pattern

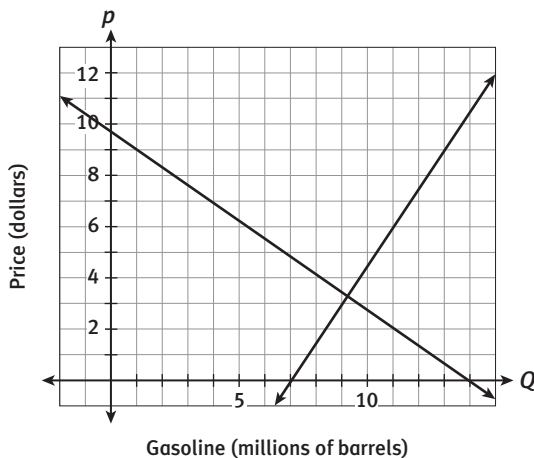
Have you ever noticed that when an item is popular and many people want to buy it, the price goes up, but items that no one wants are marked down to a lower price?

The change in an item's price and the quantity available to buy are the basis of the concept of *supply and demand* in economics. *Demand* refers to the quantity that people are willing to buy at a particular price. *Supply* refers to the quantity that the manufacturer is willing to produce at a particular price. The final price that the customer sees is a result of both supply and demand.

Suppose that during a six-month time period, the supply and demand for gasoline has been tracked and approximated by these functions, where Q represents millions of barrels of gasoline and P represents price per gallon in dollars.

- Demand function: $P = -0.7Q + 9.7$
- Supply function: $P = 1.5Q - 10.4$

To find the best balance between market price and quantity of gasoline supplied, find a **solution of a system of two linear equations**. The demand and supply functions for gasoline are graphed below.



1. **Make use of structure.** Find an approximation of the coordinates of the intersection of the supply and demand functions. Explain what the point represents.

My Notes

CONNECT TO ECONOMICS

The role of the desire for and availability of a good in determining price was described by Muslim scholars as early as the fourteenth century.

The phrase *supply and demand* was first used by eighteenth-century Scottish economists.

MATH TERMS

A point, or set of points, is a **solution of a system of equations** in two variables when the coordinates of the points make both equations true.

My Notes

TECHNOLOGY TIP

You can use a graphing calculator and its *Calculate* function to solve systems of equations in two variables.

MATH TERMS

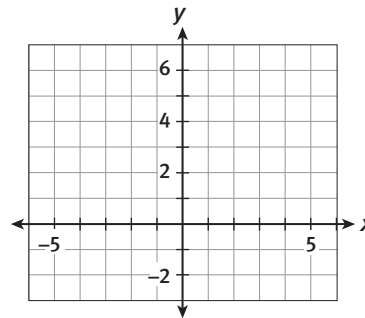
Systems of linear equations are classified by the number of solutions.

- Systems with one or many solutions are **consistent**.
- Systems with no solution are **inconsistent**.
- A system with exactly one solution is **independent**.
- A system with infinite solutions is **dependent**.

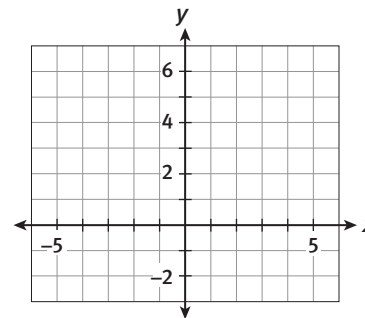
2. What problem(s) can arise when solving a system of equations by graphing?

3. **Model with mathematics.** For parts a–c, graph each system. Determine the number of solutions.

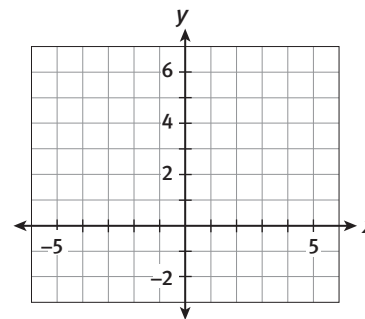
a.
$$\begin{cases} y = x + 1 \\ y = -x + 4 \end{cases}$$



b.
$$\begin{cases} y = 5 + 2x \\ y = 2x \end{cases}$$



c.
$$\begin{cases} y = 2x + 1 \\ 2y = 2 + 4x \end{cases}$$



d. Graphing two linear equations illustrates the relationships of the lines. Classify the systems in parts a–c as **consistent** and **independent**, consistent and **dependent**, or **inconsistent**.

Check Your Understanding

4. Describe how you can tell whether a system of two equations is independent and consistent by looking at its graph.
5. The graph of a system of two equations is a pair of parallel lines. Classify this system. Explain your reasoning.
6. **Make sense of problems.** A system of two linear equations is dependent and consistent. Describe the graph of the system and explain its meaning.

7. Marlon is buying a used car. The dealership offers him two payment plans, as shown in the table.

Payment Plans

Plan	Down Payment (\$)	Monthly Payment (\$)
1	0	300
2	3600	200

Marlon wants to answer this question: How many months will it take for him to have paid the same amount using either plan? Work with your group on parts a through f and determine the answer to Marlon's question.

- a. Write an equation that models the amount y Marlon will pay to the dealership after x months if he chooses Plan 1.
- b. Write an equation that models the amount y Marlon will pay to the dealership after x months if he chooses Plan 2.
- c. Write the equations as a system of equations.

My Notes

CONNECT TO PERSONAL FINANCE

A down payment is an initial payment that a customer makes when buying an expensive item, such as a house or car. The rest of the cost is usually paid in monthly installments.

DISCUSSION GROUP TIP

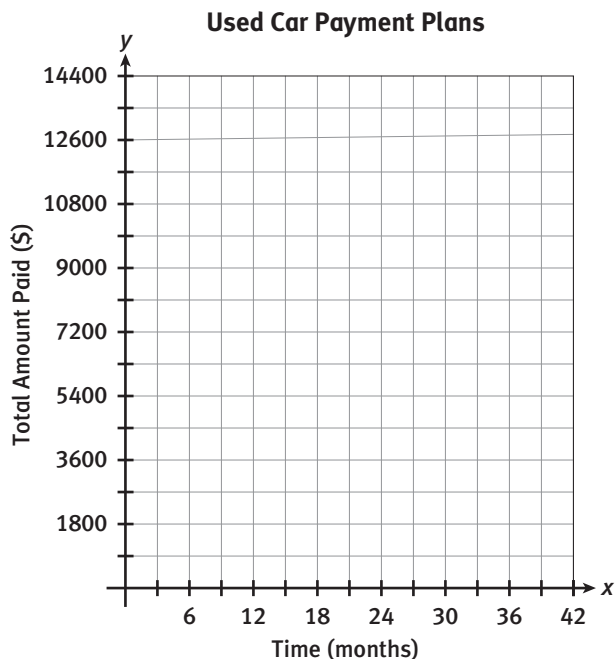
As you work with your group, review the problem scenario carefully and explore together the information provided and how to use it to create a potential solution. Discuss your understanding of the problem and ask peers or your teacher to clarify any areas that are not clear.

My Notes

MATH TIP

When graphing a system of linear equations that represents a real-world situation, it is a good practice to label each line with what it represents. In this case, you can label the lines *Plan 1* and *Plan 2*.

d. Graph the system of equations on the coordinate grid.



e. **Reason quantitatively.** What is the solution of the system of equations? What does the solution represent in this situation?

f. In how many months will the total costs of the two plans be equal?

Check Your Understanding

8. How could you check that you solved the system of equations in Item 7 correctly?
9. If Marlon plans to keep the used car less than 3 years, which of the payment plans should he choose? Justify your answer.
10. **Construct viable arguments.** Explain how to write a system of two equations that models a real-world situation.

Lesson 3-1

Solving Systems of Two Equations in Two Variables

ACTIVITY 3

continued

Investors try to control the level of risk in their portfolios by diversifying their investments. You can solve some investment problems by writing and solving systems of equations. One algebraic method for solving a system of linear equations is called **substitution**.

Example A

During one year, Sara invested \$5000 into two separate funds, one earning 2 percent and another earning 5 percent annual interest. The interest Sara earned was \$205. How much money did she invest in each fund?

Step 1: Let x = money in the first fund and y = money in the second fund.

Write one equation to represent the amount of money invested.
Write another equation to represent the interest earned.

$$\begin{aligned} x + y &= 5000 && \text{The money invested is \$5000.} \\ 0.02x + 0.05y &= 205 && \text{The interest earned is \$205.} \end{aligned}$$

Step 2: Use substitution to solve this system.

$$\begin{aligned} x + y &= 5000 && \text{Solve the first equation for } y. \\ y &= 5000 - x \\ 0.02x + 0.05(5000 - x) &= 205 && \text{Substitute for } y \text{ in the second} \\ &&& \text{equation.} \end{aligned}$$

$$\begin{aligned} 0.02x + 250 - 0.05x &= 205 && \text{Solve for } x. \\ -0.03x &= -45 \\ x &= 1500 \end{aligned}$$

Step 3: Substitute the value of x into one of the original equations to find y .

$$\begin{aligned} x + y &= 5000 \\ 1500 + y &= 5000 && \text{Substitute 1,500 for } x. \\ y &= 3500 \end{aligned}$$

Solution: Sara invested \$1500 in the first fund and \$3500 in the second fund.

Try These A

Write your answers on notebook paper. Show your work.
Solve each system of equations, using substitution.

a. $\begin{cases} x = 25 - 3y \\ 4x + 5y = 9 \end{cases}$

b. $\begin{cases} x + 2y = 14 \\ 2y = x - 10 \end{cases}$

c. $\begin{cases} y - x = 4 \\ 3x + y = 16 \end{cases}$

d. **Model with mathematics.** Eli invested a total of \$2000 in two stocks. One stock cost \$18.50 per share, and the other cost \$10.40 per share. Eli bought a total of 130 shares. Write and solve a system of equations to find how many shares of each stock Eli bought.

My Notes

MATH TERMS

In the **substitution method**, you solve one equation for one variable in terms of another. Then substitute that expression into the other equation to form a new equation with only one variable. Solve that equation. Substitute the solution into one of the two original equations to find the value of the other variable.

MATH TIP

Check your answer by substituting the solution (1500, 3500) into the second original equation, $0.02x + 0.05y = 205$

My Notes

MATH TERMS

In the **elimination method**, you *eliminate* one variable. Multiply each equation by a number so that the terms for one variable combine to 0 when the equations are added. Then use substitution with that value of the variable to find the value of the other variable. The ordered pair is the *solution* of the system.

The *elimination method* is also called the *addition-elimination method* or the *linear combination method* for solving a system of linear equations.

11. When using substitution, how do you decide which variable to isolate and which equation to solve? Explain.

Another algebraic method for solving systems of linear equations is the **elimination method**.

Example B

A stack of 20 coins contains only nickels and quarters and has a total value of \$4. How many of each coin are in the stack?

Step 1: Let n = number of nickels and q = number of quarters.

Write one equation to represent the number of coins in the stack.
Write another equation to represent the total value.

$$\begin{array}{ll} n + q = 20 & \text{The number of coins is 20.} \\ 5n + 25q = 400 & \text{The total value is 400 cents.} \end{array}$$

Step 2: To solve this system of equations, first eliminate the n variable.

$$\begin{array}{ll} \begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \begin{array}{l} -5(n + q) = -5(20) \\ 5n + 25q = 400 \\ \hline -5n - 5q = -100 \\ 5n + 25q = 400 \\ \hline 20q = 300 \\ q = 15 \end{array} & \begin{array}{l} \text{Multiply the first equation by } -5. \\ \\ \text{Add the two equations to eliminate } n. \\ \text{Solve for } q. \end{array} \end{array}$$

Step 3: Find the value of the eliminated variable n by using the original first equation.

$$\begin{array}{ll} n + q = 20 & \\ n + 15 = 20 & \text{Substitute 15 for } q. \\ \hline n = 5 & \end{array}$$

Step 4: Check your answers by substituting into the original second equation.

$$\begin{array}{ll} 5n + 25q = 400 & \\ 5(5) + 25(15) = 400 & \text{Substitute 5 for } n \text{ and 15 for } q. \\ 25 + 375 = 400 & \\ 400 = 400 & \end{array}$$

Solution: There are 5 nickels and 15 quarters in the stack of coins.

My Notes

Learning Targets:

- Solve systems of three linear equations in three variables using substitution and Gaussian elimination.
- Formulate systems of three linear equations in three variables to model a real-world situation.

SUGGESTED LEARNING STRATEGIES: Close Reading, Vocabulary Organizer, Note Taking, Summarizing, Paraphrasing, Graphic Organizer, Group Presentation, Think Aloud, Identify a Subtask

Sometimes a situation has more than two pieces of information. For these more complex problems, you may need to solve equations that contain three variables.

Read and discuss the material on this page with your group before you move on to Example A on the next page. Use your discussions to clarify the meaning of mathematical concepts and other language used to describe the information. With your group or your teacher, review background information that will be useful in applying concepts to the Example.

In Bisbee, Arizona, an old mining town, you can buy souvenir nuggets of gold, silver, and bronze. For \$20, you can buy any of these mixtures of nuggets: 14 gold, 20 silver, and 24 bronze; 20 gold, 15 silver, and 19 bronze; or 30 gold, 5 silver, and 13 bronze. What is the monetary value of each souvenir nugget?

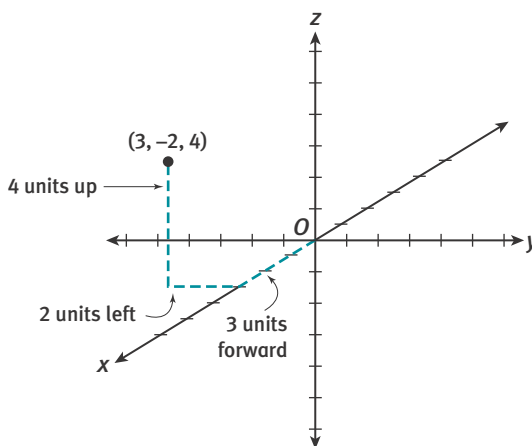
The problem above represents a system of linear equations in three variables. The system can be represented with these equations.

$$\begin{cases} 14g + 20s + 24b = 20 \\ 20g + 15s + 19b = 20 \\ 30g + 5s + 13b = 20 \end{cases}$$

Although it is possible to solve systems of equations in three variables by graphing, it can be difficult.

Just as the ordered pair (x, y) is a solution of a system in two variables, the **ordered triple** (x, y, z) is a solution of a system in three variables. Ordered triples are graphed in three-dimensional coordinate space.

The point $(3, -2, 4)$ is graphed below.



Lesson 3-2

Solving Systems of Three Equations in Three Variables

ACTIVITY 3

continued

You can use the substitution method to solve systems of equations in three variables.

Example A

Solve this system using substitution.

$$\begin{cases} 2x + 7y + z = -53 \\ -2x + 3y + z = -13 \\ 6x + 3y + z = -45 \end{cases}$$

Step 1: Solve the first equation for z .

$$\begin{aligned} 2x + 7y + z &= -53 \\ z &= -2x - 7y - 53 \end{aligned}$$

Step 2: Substitute the expression for z into the second equation. Then solve for y .

$$\begin{aligned} -2x + 3y + z &= -13 \\ -2x + 3y + (-2x - 7y - 53) &= -13 && \text{Substitute } -2x - 7y - 53 \text{ for } z. \\ -4x - 4y - 53 &= -13 && \text{Solve for } y. \\ -4y &= 4x + 40 \\ y &= -x - 10 \end{aligned}$$

Step 3: Use substitution to solve the third equation for x .

$$\begin{aligned} 6x + 3y + z &= -45 \\ 6x + 3y + (-2x - 7y - 53) &= -45 && \text{Substitute } -2x - 7y - 53 \text{ for } z. \\ 4x - 4y - 53 &= -45 \\ 4x - 4y &= 8 \\ 4x - 4(-x - 10) &= 8 && \text{Substitute } -x - 10 \text{ for } y. \\ 4x + 4x + 40 &= 8 && \text{Solve for } x. \\ 8x &= -32 \\ x &= -4 \end{aligned}$$

Step 4: Solve the last equation from Step 2 for y .

$$\begin{aligned} y &= -x - 10 \\ y &= -(-4) - 10 && \text{Substitute } -4 \text{ for } x. \\ y &= -6 \end{aligned}$$

Step 5: Solve the last equation from Step 1 for z .

$$\begin{aligned} z &= -2x - 7y - 53 \\ z &= -2(-4) - 7(-6) - 53 && \text{Substitute } -4 \text{ for } x \text{ and } -6 \text{ for } y. \\ z &= 8 + 42 - 53 \\ z &= -3 \end{aligned}$$

Solution: The solution of the system is $(-4, -6, -3)$.

My Notes

MATH TIP

As a final step, check your ordered triple solution in one of the original equations to be sure that your solution is correct.

My Notes

Try These A

Solve each system of equations using substitution. Show your work.

a.
$$\begin{cases} x + 4y + z = 3 \\ 2x + y + z = 11 \\ 4x + y + 2z = 23 \end{cases}$$

b.
$$\begin{cases} 3x + y + z = 5 \\ x + 2y - 3z = 15 \\ 2x - y + z = 2 \end{cases}$$

MATH TERMS

When using **Gaussian elimination** to solve a system of three equations in the variables x , y , and z , you start by eliminating x from the second and third equations. Then eliminate y from the third equation. The third equation now has a single variable, z ; solve the third equation for z . Then use the value of z to solve the second equation for y . Finally, use the values of y and z to solve the first equation for x .

Another method of solving a system of three equations in three variables is called **Gaussian elimination**. This method has two main parts. The first part involves eliminating variables from the equations in the system. The second part involves solving for the variables one at a time.

Example B

Solve this system using Gaussian elimination.

$$\begin{cases} 2x + y - z = 4 \\ -2x + y + 2z = 6 \\ x + 2y + z = 11 \end{cases}$$

Step 1: Use the first equation to eliminate x from the second equation.

$$\begin{array}{r} 2x + y - z = 4 \\ -2x + y + 2z = 6 \\ \hline 2y + z = 10 \end{array}$$

$$\begin{cases} 2x + y - z = 4 \\ 2y + z = 10 \\ x + 2y + z = 11 \end{cases}$$

Add the first and second equations.

Replace the second equation in the system with $2y + z = 10$.

Step 2: Use the first equation to eliminate x from the third equation.

$$\begin{array}{r} 2x + y - z = 4 \\ -2(x + 2y + z) = -2(11) \\ \hline 2x + y - z = 4 \\ -2x - 4y - 2z = -22 \\ \hline -3y - 3z = -18 \end{array}$$

Multiply the third equation by -2 .

Add the equations to eliminate x .

$$\begin{cases} 2x + y - z = 4 \\ 2y + z = 10 \\ -3y - 3z = -18 \end{cases}$$

Replace the third equation in the system with $-3y - 3z = -18$.

Lesson 3-2

Solving Systems of Three Equations in Three Variables

ACTIVITY 3

continued

Step 3: Use the second equation to eliminate y from the third equation.

$$\begin{array}{l} \left. \begin{array}{l} 3(2y + z) = 3(10) \\ 2(-3y - 3z) = 2(-18) \end{array} \right\} \begin{array}{l} \text{Multiply the second equation by} \\ \text{3 and the third equation by 2.} \end{array} \\ \hline \begin{array}{l} 6y + 3z = 30 \\ -6y - 6z = -36 \\ \hline -3z = -6 \end{array} \end{array} \quad \begin{array}{l} \text{Add the equations to eliminate } y. \end{array}$$

$$\left\{ \begin{array}{l} 2x + y - z = 4 \\ 2y + z = 10 \\ -3z = -6 \end{array} \right. \quad \begin{array}{l} \text{Replace the third equation in the} \\ \text{system with } -3z = -6. \end{array}$$

Step 4: Solve the third equation for z .

$$\begin{array}{l} -3z = -6 \\ z = 2 \end{array}$$

Step 5: Solve the second equation for y .

$$\begin{array}{l} 2y + z = 10 \\ 2y + 2 = 10 \quad \text{Substitute 2 for } z. \\ \hline 2y = 8 \\ y = 4 \end{array}$$

Step 6: Solve the first equation for x .

$$\begin{array}{l} 2x + y - z = 4 \\ 2x + 4 - 2 = 4 \quad \text{Substitute 4 for } y \text{ and 2 for } z. \\ \hline 2x + 2 = 4 \\ 2x = 2 \\ x = 1 \end{array}$$

Solution: The solution of the system is $(1, 4, 2)$.

Try These B

- a. Solve this system of equations using Gaussian elimination. Show your work.

$$\left\{ \begin{array}{l} 2x + y - z = -2 \\ x + 2y + z = 11 \\ -2x + y + 2z = 15 \end{array} \right.$$

1. Work with a partner or with your group. Make a flowchart on notebook paper that summarizes the steps for solving a system of three equations in three variables by using either substitution or Gaussian elimination. As you prepare your flowchart to present to the class, remember to use words and graphics that will help your classmates understand the steps. Also, be careful to communicate mathematical terms correctly to describe the application of mathematical concepts.

My Notes

CONNECT TO MATH HISTORY

The method of Gaussian elimination is named for the German mathematician Carl Friedrich Gauss (1777–1855), who used a version of it in his calculations. However, the first known use of Gaussian elimination was a version used in a Chinese work called *Nine Chapters of the Mathematical Art*, which was written more than 2000 years ago. It shows how to solve a system of linear equations involving the volume of grain yielded from sheaves of rice.

My Notes

A farmer plans to grow corn, soybeans, and wheat on his farm. Let c represent the number of acres planted with corn, s represent the number of acres planted with soybeans, and w represent the number of acres planted with wheat.

- The farmer has 500 acres to plant with corn, soybeans, and wheat. Write an equation in terms of c , s , and w that models this information.
- Growing an acre of corn costs \$390, an acre of soybeans costs \$190, and an acre of wheat costs \$170. The farmer has a budget of \$119,000 to spend on growing the crops. Write an equation in terms of c , s , and w that models this information.
- The farmer plans to grow twice as many acres of wheat as acres of corn. Write an equation in terms of c and w that models this information.
- Write your equations from Items 3–5 as a system of equations.
- Make sense of problems.** Solve the system of equations. Write the solution as an ordered triple of the form (c, s, w) .

MATH TIP

Determine the reasonableness of your solution. Does your answer make sense in the context of the problem?

My Notes

WRITING MATH

You can name a matrix by using a capital letter.

READING MATH

To say the dimensions of matrix A , read 2×3 as "2 by 3."

MATH TERMS

A **matrix** (plural: matrices) is a rectangular array of numbers arranged in rows and columns inside brackets.

The **dimensions of a matrix** are the number of rows and the number of columns, indicated by $m \times n$, where m is the number of rows and n is the number of columns.

The **entries** of a matrix are the numbers in the matrix.

Learning Targets:

- Add, subtract, and multiply matrices.
- Use a graphing calculator to perform operations on matrices.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Note Taking, Close Reading, Summarizing, Paraphrasing, Discussion Groups, Work Backward

A **matrix**, such as matrix A below, is a rectangular arrangement of numbers written inside brackets.

$$A = \begin{bmatrix} 2 & 4 & 5 \\ -3 & 8 & -2 \end{bmatrix}$$

The **dimensions of a matrix** give its number of rows and number of columns. A matrix with m rows and n columns has dimensions $m \times n$. Matrix A has 2 rows and 3 columns, so its dimensions are 2×3 .

The numbers in a matrix are called **entries**. The address of an entry gives its location in the matrix. To write the address of an entry, write the lowercase letter used to name the matrix, and then write the row number and column number of the entry as subscripts. The address a_{12} indicates the entry in matrix A in the first row and second column, so a_{12} is 4.

In the next lesson, you will learn how to use matrices to solve systems of equations.

Use these matrices to answer Items 1–3.

$$B = \begin{bmatrix} 3 & -6 \\ 8 & 10 \\ -4 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 14 & 40 \\ 26 & 30 \end{bmatrix}$$

1. What are the dimensions of each matrix?
2. **Make use of structure.** Write the entry indicated by each address.
 - a. b_{31}
 - b. b_{12}
 - c. c_{21}
 - d. c_{22}
3. What is the address of the entry 8 in matrix B ? Explain.

Lesson 3-3

Matrix Operations

ACTIVITY 3

continued

You can input a matrix into a graphing calculator using the steps below.

Step 1: Go to the Matrix menu. To do this, press $\boxed{2\text{nd}}$, and then press the key with MATRIX printed above it.

Step 2: Use the right arrow key to select the Edit submenu.

Step 3: Move the cursor next to the name of one of the matrices and press $\boxed{\text{ENTER}}$ to select it.

Step 4: Enter the correct dimensions for the matrix.

Step 5: Enter the entries of the matrix. To save the matrix, press $\boxed{2\text{nd}}$, and then press the key with QUIT printed above it.

4. Input each matrix into a graphing calculator.

a. $A = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}$

b. $B = \begin{bmatrix} 3 & -2 & 5 \\ 6 & 0 & -4 \end{bmatrix}$

If two matrices have the same dimensions, you can add or subtract them by adding or subtracting their corresponding entries.

Example A

Find each matrix sum or difference.

$$C = \begin{bmatrix} 2 & 8 & 10 \\ -3 & 1 & -5 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 0 & -2 \\ 8 & 7 & -4 \end{bmatrix}$$

a. Find $C + D$.

$$C + D = \begin{bmatrix} 2+5 & 8+0 & 10+(-2) \\ -3+8 & 1+7 & -5+(-4) \end{bmatrix} = \begin{bmatrix} 7 & 8 & 8 \\ 5 & 8 & -9 \end{bmatrix}$$

b. Find $C - D$.

$$C - D = \begin{bmatrix} 2-5 & 8-0 & 10-(-2) \\ -3-8 & 1-7 & -5-(-4) \end{bmatrix} = \begin{bmatrix} -3 & 8 & 12 \\ -11 & -6 & -1 \end{bmatrix}$$

Try These A

Find each matrix sum or difference.

$$E = \begin{bmatrix} 6 & -2 \\ 9 & 4 \end{bmatrix}$$

$$F = \begin{bmatrix} -1 & 5 \\ 10 & 7 \end{bmatrix}$$

a. $E + F$

b. $E - F$

c. $F - E$

My Notes

TECHNOLOGY TIP

When entering a negative number as an entry in a matrix, be sure to use the negative key to enter the negative sign, not the subtraction key.

TECHNOLOGY TIP

To add the matrices on a graphing calculator, first input both matrices. Then select [C] from the Names submenu of the Matrix menu. Press $\boxed{+}$. Then select [D] from the Names submenu of the Matrix menu. Your screen should now show [C]+[D]. Press $\boxed{\text{ENTER}}$ to show the sum.

My Notes

MATH TIP

Recall that the Commutative Property of Addition states that $a + b = b + a$ for any real numbers a and b .

MATH TIP

The inner dimensions of two matrices indicate whether their product is defined.

$$n \times \quad \quad \times p$$

The outer dimensions indicate the dimensions of the matrix product.

$$\times m \quad m \times$$

Check Your Understanding

- How is a matrix similar to a table?
- Express regularity in repeated reasoning.** Make a conjecture about whether matrix addition is commutative. Then provide an example that supports your conjecture.
- Explain why you cannot subtract these two matrices.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -6 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & -2 \\ 5 & -6 \\ 6 & -10 \end{bmatrix}$$

- Two matrices are additive inverses if each entry in their sum is 0. What is the additive inverse of the matrix shown below? Explain how you determined your answer.

$$\begin{bmatrix} 7 & -2 \\ 0 & 4 \end{bmatrix}$$

You can also find the product of two matrices A and B if the number of columns in A is equal to the number of rows in B . For example, the dimensions of matrix A below are 3×2 , and the dimensions of matrix B are 2×1 . The matrix product AB is defined because A has 2 columns and B has 2 rows.

$$A = \begin{bmatrix} 2 & 8 \\ -7 & 5 \\ 1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

The product of an $n \times m$ matrix and an $m \times p$ matrix is an $n \times p$ matrix. Because A above is a 3×2 matrix and B is a 2×1 matrix, the product AB is a 3×1 matrix.

To find the entry in row i and column j of the product AB , find the sum of the products of consecutive entries in row i of matrix A and column j of matrix B . To see what this means, take a look at the next example.

My Notes

Example B

Find the matrix product AB .

$$A = \begin{bmatrix} 1 & 5 & -4 \\ 3 & -2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 \\ 3 & -1 \\ 0 & -2 \end{bmatrix}$$

Step 1: Determine whether AB is defined.

A is a 2×3 matrix, and B is a 3×2 matrix, so AB is defined.
 A has 2 rows and B has 2 columns, so AB is a 2×2 matrix.

Step 2: Find the entry in row 1, column 1 of AB .

Use row 1 of A and column 1 of B . Multiply the first entries, the second entries, and the third entries. Then add the products.

$$1(2) + 5(3) + (-4)(0) = 17$$

$$AB = \begin{bmatrix} 1 & 5 & -4 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & - \\ - & - \end{bmatrix}$$

Step 3: Find the entry in row 1, column 2 of AB .

Use row 1 of A and column 2 of B .

$$1(4) + 5(-1) + (-4)(-2) = 7$$

$$AB = \begin{bmatrix} 1 & 5 & -4 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 7 \\ - & - \end{bmatrix}$$

Step 4: Find the entry in row 2, column 1 of AB .

Use row 2 of A and column 1 of B .

$$3(2) + (-2)(3) + 2(0) = 0$$

$$AB = \begin{bmatrix} 1 & 5 & -4 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 7 \\ 0 & - \end{bmatrix}$$

Step 5: Find the entry in row 2, column 2 of AB .

Use row 2 of A and column 2 of B .

$$3(4) + (-2)(-1) + 2(-2) = 10$$

$$AB = \begin{bmatrix} 1 & 5 & -4 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 7 \\ 0 & 10 \end{bmatrix}$$

Solution: $AB = \begin{bmatrix} 17 & 7 \\ 0 & 10 \end{bmatrix}$

TECHNOLOGY TIP

To multiply the matrices on a graphing calculator, first input both matrices. Then select [A] from the Names submenu of the Matrix menu. Press \boxed{x} . Then select [B] from the Names submenu of the Matrix menu. Your screen should now show [A]*[B]. Press $\boxed{\text{ENTER}}$ to show the product.

My Notes

Try These B

Find each matrix product if it is defined.

$$C = \begin{bmatrix} 1 & 4 \\ -7 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 8 \\ -4 & -5 \end{bmatrix} \quad E = \begin{bmatrix} 8 & 5 & -1 \\ -4 & 4 & -9 \end{bmatrix}$$

a. CD b. CE c. ED

Check Your Understanding

9. Is matrix multiplication commutative? Provide an example that supports your answer.
10. The matrix product RS is a 3×4 matrix. If R is a 3×2 matrix, what are the dimensions of S ? Explain your answer.
11. **Critique the reasoning of others.** Rebekah made an error when finding the matrix product KL . Her work is shown below. What mistake did Rebekah make? What is the correct matrix product?

$$K = \begin{bmatrix} 2 & 8 \\ -4 & -2 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 5 \\ 0 & -3 \end{bmatrix}$$

$$KL = \begin{bmatrix} 2(1) & 8(5) \\ -4(0) & -2(-3) \end{bmatrix} = \begin{bmatrix} 2 & 40 \\ 0 & 6 \end{bmatrix}$$

LESSON 3-3 PRACTICE

Use these matrices to answer Items 12–17.

$$A = \begin{bmatrix} 3 & -6 & 1 \\ -8 & 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 5 & -7 \\ -3 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 10 \\ -1 & -3 \end{bmatrix}$$

12. What are the dimensions of A ?
13. **Look for and make use of structure.** What is the entry with the address b_{12} ?
14. Find $B + C$.
15. Find $C - B$.
16. Find AB if it is defined.
17. Find BC if it is defined.

Learning Targets:

- Solve systems of two linear equations in two variables by using graphing calculators with matrices.
- Solve systems of three linear equations in three variables by using graphing calculators with matrices.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Note Taking, Discussion Groups, Marking the Text, Debriefing, Identify a Subtask, Create Representations, Look for a Pattern

A **square matrix** is a matrix with the same number of rows and columns. A **multiplicative identity matrix** is a square matrix in which all entries along the main diagonal are 1 and all other entries are 0. The main diagonal of a square matrix is the diagonal from the upper left to the lower right. A multiplicative identity matrix is often called an identity matrix and is usually named I .

A 3×3 identity matrix is shown below. The entries in blue are on the main diagonal.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The product of a square matrix and its **multiplicative inverse matrix** is an identity matrix I . The multiplicative inverse of matrix A is often called the inverse of A and may be named as A^{-1} . So, by definition, A^{-1} is the inverse of A if $A \cdot A^{-1} = I$.

Check Your Understanding

- 1. Construct viable arguments.** Explain why $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is *not* an identity matrix.

Use these matrices and a graphing calculator to answer Items 2–4.

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1.5 & -2 \\ -0.5 & 1 \end{bmatrix}$$

2. Multiply A by a 2×2 identity matrix. Describe the relationship between the matrix product AI and A .
3. Is B the inverse of A ? Explain.
4. Is A the inverse of B ? Explain.

You can use matrices and inverse matrices to solve systems of linear equations.

My Notes

MATH TERMS

A **square matrix** is a matrix in which the number of rows equals the number of columns.

A **multiplicative identity matrix**, or identity matrix, is a square matrix in which all entries are 0 except the entries along the main diagonal, all of which are 1.

An identity matrix is the product of a square matrix and its **multiplicative inverse matrix**, or inverse matrix.

MATH TIP

Not all square matrices have inverses. But if a square matrix has an inverse, it must be a square matrix.

My Notes

MATH TIP

The second equation in the system, $x - 4y = -2$, can be written as $1x + (-4)y = -2$, which more clearly shows that the coefficient of x is 1 and the coefficient of y is -4 .

MATH TERMS

A **matrix equation** is an equation of the form $AX = B$.

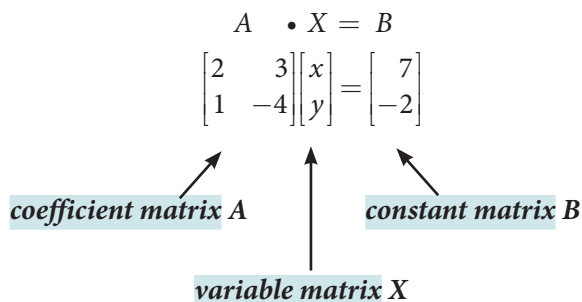
- A is the **coefficient matrix**, the matrix formed by the coefficients of the system of equations.
- X is the **variable matrix**, a column matrix that represents all the variables of the system of equations.
- B is the **constant matrix**, a column matrix representing all the constants of the systems of equations.

TECHNOLOGY TIP

If the entries in the inverse matrix are decimals, try converting them to fractions by pressing $\boxed{\text{MATH}}$ and selecting 1: \blacktriangleright Frac.

The first step in solving a system of linear equations by using matrices is to write the system as a **matrix equation**. The diagram shows how to write the

system $\begin{cases} 2x + 3y = 7 \\ x - 4y = -2 \end{cases}$ as a matrix equation.



To solve a matrix equation $AX = B$ for X , you use a process similar to what you would use when solving the regular equation $ax = b$ for x . To solve $ax = b$, you could multiply both sides of the equation by the multiplicative inverse of a .

Likewise, to solve the matrix equation $AX = B$, you can multiply both sides of the equation by the multiplicative inverse matrix of A . Thus, the solution of $AX = B$ is $X = A^{-1}B$. You can use a graphing calculator to help you find A^{-1} .

In Items 5–7, use the matrix equation $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$.

5. To solve for X , you first need to find A^{-1} . Input matrix A , the coefficient matrix, into a graphing calculator. Then select $[A]$ from the Names submenu of the Matrix menu. Then press $\boxed{x^{-1}}$. Your screen should now show $[A]^{-1}$. Press $\boxed{\text{ENTER}}$ to show the inverse matrix. What is A^{-1} ?

6. Find the matrix product $A^{-1}B$.

Lesson 3-4

Solving Matrix Equations

ACTIVITY 3

continued

7. **Make sense of problems.** What are the values of x and y in the variable matrix X ? How do you know?

Example A

The hourly cost to a police department of using a canine team depends on the hourly cost x in dollars of using a dog and the hourly salary y of a handler. The hourly cost for a team of three dogs and two handlers is \$82, and the hourly cost for a team of four dogs and four handlers is \$160. The

system $\begin{cases} 3x + 2y = 82 \\ 4x + 4y = 160 \end{cases}$ models this situation. Use a matrix equation to

solve the system, and explain what the solution means.

Step 1: Use the system to write a matrix equation.

$$\begin{bmatrix} 3 & 2 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 82 \\ 160 \end{bmatrix}$$

Step 2: Enter the coefficient matrix A and the constant matrix B into a graphing calculator.

MATRIX[A] 2 × 2
[3 2]
[4 4]

MATRIX[B] 2 × 1
[82]
[160]

Step 3: Use the calculator to find $A^{-1}B$.

$[A]^{-1} * [B]$
[[2]
[38]]

Step 4: Identify and interpret the solution of the system.

Solution: The matrix product $A^{-1}B$ is equal to the variable matrix X , so $x = 2$ and $y = 38$. The solution of the system is $(2, 38)$.

The solution shows that the hourly cost of using a dog is \$2 and the hourly salary of a handler is \$38.

Try These A

Write a matrix equation to model each system. Then use the matrix equation to solve the system.

a. $\begin{cases} 2x + y = 8 \\ 5x + 6y = 13 \end{cases}$

b. $\begin{cases} 6x - 3y = -18 \\ 2x + 4y = 34 \end{cases}$

c. $\begin{cases} x - 2y = -23 \\ 3x + 3y = 21 \end{cases}$

My Notes

MATH TIP

Before you can write a matrix equation to model a system of two linear equations in two variables, each equation in the system must be written in standard form $Ax + By = C$, where A , B , and C are real numbers.

My Notes

8. What is an advantage of using a graphing calculator to solve a system of two linear equations in two variables as opposed to solving the system by making a hand-drawn graph?

DISCUSSION GROUP TIPS

As you work in groups, review the problem scenario carefully and explore together the information provided and how to use it to create potential solutions. Discuss your understanding of the problem and ask peers or your teacher to clarify any areas that are not clear in order to aid comprehension and to ensure understanding of all group members' ideas.

You can also use a matrix equation to solve a system of three linear equations in three variables.

Use this information to complete Items 9–15 with your group. Karen makes handmade greeting cards and sells them at a local store. The cards come in packs of 4 for \$11, 6 for \$15, or 10 for \$20. Last month, the store sold 16 packs containing 92 of Karen's cards for a total of \$223. The following system models this situation where x is the number of small packs, y is the number of medium packs, and z is the number of large packs.

$$\begin{cases} x + y + z = 16 \\ 4x + 6y + 10z = 92 \\ 11x + 15y + 20z = 223 \end{cases}$$

9. If you were to model the system with a matrix equation, what would be the dimensions of the coefficient matrix? How do you know?

10. **Model with mathematics.** Write a matrix equation to model the system.

Lesson 3-4

Solving Matrix Equations

ACTIVITY 3

continued

- 11. Use appropriate tools strategically.** Use a graphing calculator to find A^{-1} , the inverse of the coefficient matrix.
- 12.** Use a graphing calculator to find $A^{-1}B$.
- 13.** Find the solution of the system of equations and explain the meaning of the solution.
- 14.** How can you check that you found the solution of the system correctly?
- 15.** Work with your group. Compare and contrast using a matrix equation to solve a system of two linear equations in two variables with using a matrix equation to solve a system of three linear equations in three variables.

My Notes

DISCUSSION GROUP TIP

As you listen to the group discussion, take notes to aid comprehension and to help you describe your own ideas to others in your group. Ask questions to clarify ideas and to gain further understanding of key concepts.

My Notes

Check Your Understanding

16. A system of equations and the matrix equation that models it are shown below. Find AX , the product of the coefficient matrix and the variable matrix of the matrix equation. What is the relationship between AX and the system of equations?

$$\begin{cases} 3x + 2y = 18 \\ 2x + 4y = 20 \end{cases} \qquad \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 20 \end{bmatrix}$$

17. **Critique the reasoning of others.** Doug incorrectly solved the matrix equation in Item 17 by finding the matrix product $\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 18 \\ 20 \end{bmatrix}$.

What mistake did Doug make? What should he have done instead?

18. What happens when you try to solve the system $\begin{cases} 2x + y = 6 \\ 4x + 2y = 16 \end{cases}$ by writing and solving a matrix equation? What do you think this result indicates about the system? Confirm your answer by graphing the system and using the graph to classify the system.

LESSON 3-4 PRACTICE

19. Use a graphing calculator to find the inverse of the matrix $\begin{bmatrix} 2 & 8 \\ 5 & 4 \end{bmatrix}$.

For Items 20–21, write a matrix equation to model each system. Then use the matrix equation to solve the system.

20. $\begin{cases} 2x + 4y = 22 \\ -3x + 2y = 7 \end{cases}$ 21. $\begin{cases} x + y + z = 11 \\ 2x + 8y + 3z = 80 \\ 4x - 6y + 7z = -62 \end{cases}$

CONNECT TO ECONOMICS

The euro is the unit of currency used in many of the nations in the European Union. The British pound is the unit of currency used in the United Kingdom.

22. **Model with mathematics.** Steve has 2 euros and 4 British pounds worth a total of \$9.10. Emily has 3 euros and 1 British pound worth a total of \$5.55.
- Write a system of equations to model this situation, where x represents the value of 1 euro in dollars and y represents the value of 1 British pound in dollars.
 - Write the system of equations as a matrix equation.
 - Use the matrix equation to solve the system. Then interpret the solution.
23. Which solution method for solving systems of equations do you find easiest to use? Which method do you find most difficult to use? Explain why.

ACTIVITY 3 PRACTICE

Write your answers on notebook paper.
 Show your work.

Lesson 3-1

1. Solve the system by graphing.

$$\begin{cases} y = -3x + 6 \\ y = -\frac{1}{3}x - 2 \end{cases}$$

2. Solve the system using substitution.

$$\begin{cases} 5y - x = -5 \\ 7y - x = -23 \end{cases}$$

3. Solve the system using elimination.

$$\begin{cases} 2x - 5y = 8 \\ x - 3y = -1 \end{cases}$$

4. The system of equations $\begin{cases} 2x + 3y = 7 \\ 10x + cy = 3 \end{cases}$ has

solutions for all values of c except:

- A. -15 B. -3
 C. 10 D. 15

5. a. Graph the system $\begin{cases} y = -2x - 1 \\ 3y = -3 - 6x \end{cases}$

b. Classify the system, and tell how many solutions it has.

6. Mariana had a \$20 gift card to an online music store. She spent the entire amount on songs, which cost \$1 each, and music videos, which cost \$2 each. Mariana bought five more songs than music videos. Write and solve a system of equations to find the number of songs and the number of music videos Mariana bought.

7. A chemist needs to mix a 2% acid solution and a 10% acid solution to make 600 milliliters of a 5% acid solution. Write and solve a system of equations to find the volume of the 2% solution and the volume of the 10% solution that the chemist will need.

Lesson 3-2

8. Solve the system using substitution.

$$\begin{cases} x + y + z = 6 \\ 2x + y + 2z = 14 \\ 3x + 3y + z = 8 \end{cases}$$

9. Solve the system using Gaussian elimination.

$$\begin{cases} 2x + 4y + z = 31 \\ -2x + 2y - 3z = -9 \\ x + 3y + 2z = 21 \end{cases}$$

10. A snack company plans to sell a mixture of peanut butter, grape jelly, and granola as a sandwich spread. The table gives information about each ingredient.

Ingredient	Calories per Ounce	Grams of Fat per Ounce
Peanut butter	168	14
Grape jelly	71	0
Granola	132	12

An 18-ounce jar of the sandwich spread will have a total of 2273 calories and 150 grams of fat. Write and solve a system of equations to find the number of ounces of peanut butter, grape jelly, and granola in each jar.

11. A small furniture factory makes three types of tables: coffee tables, dining tables, and end tables. The factory needs to make 54 tables each day. The number of dining tables made per day should equal the number of coffee tables and end tables combined. The number of coffee tables made each day should be three more than the number of end tables. Write and solve a system of equations to find the number of tables of each type the factory should make each day.

12. Can you solve this system? Explain.

$$\begin{cases} x + 2y + 3z = 8 \\ 3x + 4y + 5z = 10 \end{cases}$$

Lesson 3-3

Use the given matrices for Items 13–20.

$$A = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ -2 & 4 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 3 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

13. What are the dimensions of matrix A ?
14. What is the entry with the address c_{13} ?
15. Find $A + D$.
16. Find $B - E$.
17. Find ED if it is defined.
18. Find AC if it is defined.
19. Find AB if it is defined.
20. Let P equal the matrix product BA . Which expression gives the value of P_{12} ?
 - A. $-2(3) + 5(-1)$
 - B. $1(3) + 5(1)$
 - C. $4(0) + (-2)(2)$
 - D. $4(0) + 1(2)$
21. Explain how to determine whether the product of two matrices is defined and how to determine the dimensions of a product matrix.

Lesson 3-4

22. Find the inverse of each matrix.
 - a. $\begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix}$
 - b. $\begin{bmatrix} -2 & -1 \\ 2 & 3 \end{bmatrix}$
23. Are these matrices inverses of each other? Explain.

$$A = \begin{bmatrix} 0 & 4 \\ 1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 1 \\ 0.25 & 0 \end{bmatrix}$$

24. Write the system of equations represented by the matrix equation below. Then solve the matrix equation.

$$\begin{bmatrix} -3 & -2 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

25. Write a matrix equation to model the system. Then use the matrix equation to solve the system.

$$\begin{cases} 3x + 2y - 7z = -29 \\ 4x - 6y + 5z = -19 \\ 8x + y - 4z = -30 \end{cases}$$

26. Guillermo bought ground beef and ground pork for a party. The beef costs \$3.48/lb and the pork costs \$2.64/lb. Guillermo bought 6 pounds of meat for a total of \$19.62. Write a system of equations that can be used to determine how many pounds of each type of meat Guillermo bought. Then use a matrix equation to solve the system.
27. Dean, John, and Andrew sold key chains, mugs, and gift wrap for a school fundraiser. The table below shows the number of items that each person sold and the amount of money collected from the sales. Write a matrix equation that can be used to find the price for each item in the table. Then solve the equation to find the prices.

	Key Chains	Mugs	Gift Wrap	Amount of Sales
Dean	3	4	5	\$87.50
John	1	1	2	\$31.50
Andrew	2	0	1	\$17.00

MATHEMATICAL PRACTICES

Look For and Make Use of Structure

28. Compare and contrast solving an equation of the form $ax = b$ for x with solving a matrix equation of the form $AX = B$ for X .

A gaming manufacturing company is developing a new gaming system. In addition to a game console, the company will also produce an optional accessory called a Jesture that allows users to communicate with the game console by using gestures and voice commands.

Solve the following problems about the gaming system. Show your work.

- The company plans to sell the video game console at a loss in order to increase its sales. It will make up for the loss from profits made from the sales of games for the system. The company will lose \$50 for each console it sells and earn a profit of \$15 for each game sold for the system.
 - Write an equation that can be used to determine t , the total amount the company will earn from a customer who buys a console and g games.
 - Graph the equation on a coordinate grid.
 - The company predicts that the average customer will buy seven games for the video game console. What is the total amount the company will earn from the average customer who buys a game console and seven games?
- To produce the new system, the company plans on using resources in two manufacturing plants. The table gives the hours needed for three tasks. For both plants combined, the company has allocated the following resources on a weekly basis: no more than 8500 hours of motherboard production, no more than 9000 hours of technical labor, and no more than 12,000 hours of general manufacturing.

Resources	Plant 1 (hours per system)	Plant 2 (hours per system)
Motherboard production	9	1
Technical labor	9	3
General manufacturing	4	8

- Write inequalities that model the constraints in this situation. Let x represent the number of gaming systems that will be made in Plant 1, and let y represent the number of gaming systems that will be made in Plant 2.
 - Graph the constraints. Shade the solution region that is common to all of the inequalities.
 - Identify an ordered pair that satisfies the constraints. Explain what the ordered pair represents in the context of the situation.
- The Jesture accessory can recognize players when they are within a certain range. The player's distance from the Jesture can vary up to 1.2 meters from the target distance of 2.4 meters.
 - Write an absolute value equation that can be used to find the extreme distances that a player can stand from the Jesture and still be recognized.
 - Solve your equation, and interpret the solutions.

4. The Jesture will come with a fitness program. The program allows players to earn fitness points depending on the number of minutes they spend on each activity. The table shows how many minutes three players spent on each activity and the total number of fitness points they earned.

Play Tester	Yoga (minutes)	Aerobics (minutes)	Jogging (minutes)	Fitness Points
Cassie	30	10	20	130
Clint	15	20	10	95
Kian	10	25	20	140

- Write a system of three equations that can be used to determine the number of points a player gets for 1 minute of each activity.
- Solve your system, and interpret the solution.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 2b, 3b, 4b)	<ul style="list-style-type: none"> Clear and accurate understanding of solving systems of equations and inequalities, and absolute value equations 	<ul style="list-style-type: none"> A functional understanding of solving systems of equations and inequalities and absolute value equations 	<ul style="list-style-type: none"> Partial understanding of solving systems of equations and inequalities and absolute value equations 	<ul style="list-style-type: none"> Little or no understanding of solving systems of equations and inequalities and absolute value equations
Problem Solving (Items 1c, 2c, 3b, 4b)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but results in a correct answer 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers 	<ul style="list-style-type: none"> No clear strategy when solving problems
Mathematical Modeling / Representations (Items 1a, 1b, 2a, 2b, 3a, 4a)	<ul style="list-style-type: none"> Fluency in representing real-world scenarios using linear equations, systems of equations and inequalities, and absolute value equations Clear and accurate understanding of creating graphs of equations and inequalities 	<ul style="list-style-type: none"> Little difficulty representing real-world scenarios using linear equations, systems of equations and inequalities, and absolute value equations Mostly accurate creation of graphs of equations and inequalities 	<ul style="list-style-type: none"> Partial understanding of how to represent real-world scenarios using linear equations, systems of equations and inequalities, and absolute value equations Partially accurate creation of graphs of equations and inequalities 	<ul style="list-style-type: none"> Little or no understanding of how to represent real-world scenarios using linear equations, systems of equations and inequalities, and absolute value equations Inaccurate or incomplete creation of graphs of equations and inequalities
Reasoning and Communication (Items 2c, 3b, 4b)	<ul style="list-style-type: none"> Ease and accuracy in explaining interpreting solutions in the context of a real-world scenario 	<ul style="list-style-type: none"> Little difficulty in explaining interpreting solutions in the context of a real-world scenario 	<ul style="list-style-type: none"> Partially correct explanations and interpretations of solutions in the context of a real-world scenario 	<ul style="list-style-type: none"> Incomplete or inaccurate explanations and interpretations of solutions in the context of a real-world scenario

Piecewise-Defined Functions

Absolutely Piece-ful

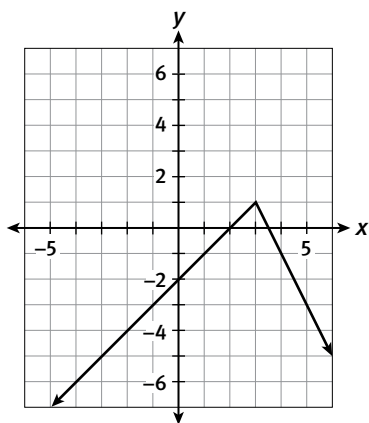
Lesson 4-1 Introduction to Piecewise-Defined Functions

Learning Targets:

- Graph piecewise-defined functions.
- Write the domain and range of functions using interval notation, inequalities, and set notation.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Quickwrite, Create Representations, Interactive Word Wall, Marking the Text, Think-Pair-Share, Discussion Groups

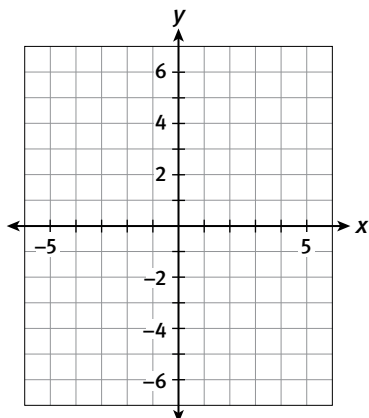
The graphs of both $y = x - 2$ for $x < 3$ and $y = -2x + 7$ for $x \geq 3$ are shown on the same coordinate grid below.



1. Work with your group on this item and on Items 2–4. Describe the graph as completely as possible.

2. **Make use of structure.** Why is the graph a function?

3. Graph $y = x^2 - 3$ for $x \leq 0$ and $y = \frac{1}{4}x + 1$ for $x > 0$ on the same coordinate grid.



My Notes

DISCUSSION GROUP TIP

As you listen to your group's discussions as you work through Items 1–4, you may hear math terms or other words that you do not know. Use your math notebook to record words that are frequently used. Ask for clarification of their meaning, and make notes to help you remember and use those words in your own communications.

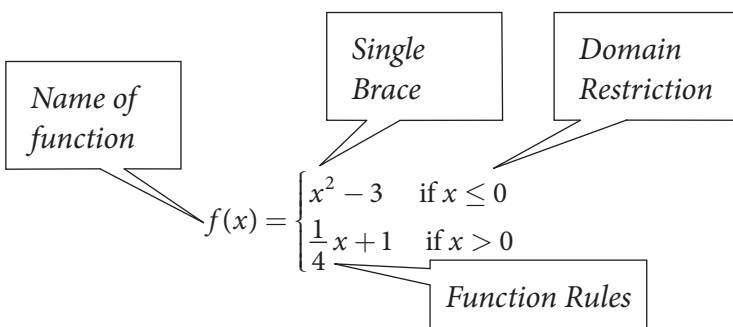
My Notes

4. Describe the graph in Item 3 as completely as possible. Why is the graph a function?

MATH TERMS

A **piecewise-defined function** is a function that is defined using different rules for the different nonoverlapping intervals of its domain.

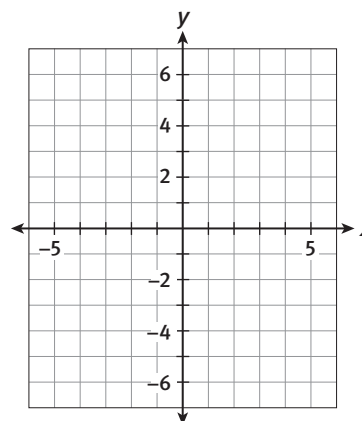
The functions in Items 1 and 3 are **piecewise-defined functions**. Piecewise-defined functions are written as follows (using the function from Item 3 as an example):



5. **Model with mathematics.** Complete the table of values. Then graph the function.

$$g(x) = \begin{cases} -2x - 2 & \text{if } x < -1 \\ x + 3 & \text{if } x \geq -1 \end{cases}$$

x	$g(x)$
-4	
-3	
-2	
-1	
0	
1	
2	



MATH TIP

A piecewise-defined function may have more than two rules. For example, consider the function below.

$$h(x) = \begin{cases} -2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

Lesson 4-1

Introduction to Piecewise-Defined Functions

ACTIVITY 4

continued

Check Your Understanding

- Critique the reasoning of others.** Look back at Item 5. Esteban says that $g(-1) = 2$. Is Esteban correct? Explain.
- Explain how to graph a piecewise-defined function.
- If a piecewise-defined function has a break, how do you know whether to use an open circle or a closed circle for the endpoints of the function's graph?

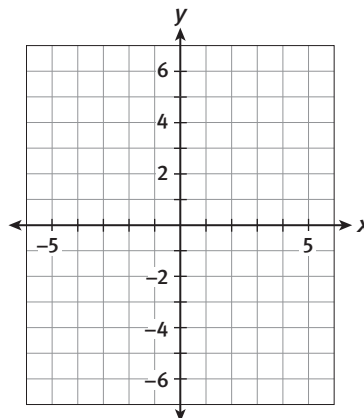
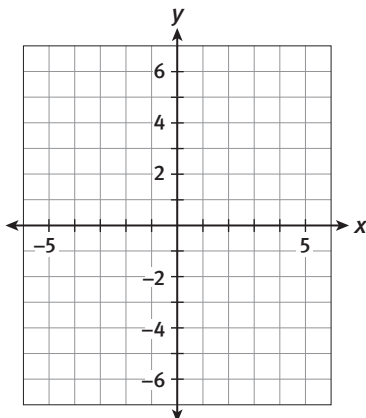
The **domain** of a piecewise-defined function consists of the union of all the domains of the individual “pieces” of the function. Likewise, the **range** of a piecewise-defined function consists of the union of all the ranges of the individual “pieces” of the function.

You can represent the domain and range of a function by using inequalities. You can also use interval notation and set notation to represent the domain and range.

- Write the domain and range of $g(x)$ in Item 5 by using:
 - inequalities
 - interval notation
 - set notation.
- Graph each function, and write its domain and range using inequalities, interval notation, and set notation. Show your work.

a.
$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 2x - 1 & \text{if } x \geq 0 \end{cases}$$

b.
$$g(x) = \begin{cases} -2x + 2 & \text{if } x < 1 \\ x - 2 & \text{if } x > 1 \end{cases}$$



My Notes

MATH TERMS

The **domain** of a function is the set of input values for which the function is defined.

The **range** of a function is the set of all possible output values for the function.

MATH TIP

Interval notation is a way of writing an interval as a pair of numbers, which represent the endpoints. For example, $2 < x \leq 6$ is written in interval notation as $(2, 6]$. Use a parenthesis if an endpoint is not included; use a bracket if an endpoint is included. In interval notation, infinity, ∞ , and negative infinity, $-\infty$, are not included as endpoints.

Set notation is a way of describing the numbers that are members, or elements, of a set. For example, $2 < x \leq 6$ is written in set notation as $\{x \mid x \in \mathbb{R}, 2 < x \leq 6\}$, which is read “the set of all numbers x such that x is an element of the real numbers and $2 < x \leq 6$.”

My Notes

WRITING MATH

You can use these symbols when writing a domain or range in set notation.

- | such that
- \in is an element of
- \mathbb{R} the real numbers
- \mathbb{Z} the integers
- \mathbb{N} the natural numbers

Check Your Understanding

11. The domain of a function is all positive integers. How could you represent this domain using set notation?
12. Explain how to use interval and set notation to represent the range $y \geq 3$.
13. What can you conclude about the graph of a piecewise-defined function whose domain is $\{x \mid x \in \mathbb{R}, x \neq 2\}$?

LESSON 4-1 PRACTICE

14. Graph each piecewise-defined function. Then write its domain and range using inequalities, interval notation, and set notation.

$$\text{a. } f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \frac{1}{2}x & \text{if } x > 0 \end{cases} \qquad \text{b. } f(x) = \begin{cases} 3x & \text{if } x < -1 \\ -x + 2 & \text{if } x \geq -1 \end{cases}$$

15. The range of a function is all real numbers greater than or equal to -5 and less than or equal to 5 . Write the range of the function using an inequality, interval notation, and set notation.
16. Evaluate the piecewise function for $x = -2$, $x = 0$, and $x = 4$.

$$g(x) = \begin{cases} -4x & \text{if } x < -2 \\ 3x + 2 & \text{if } -2 \leq x < 4 \\ x + 4 & \text{if } x \geq 4 \end{cases}$$

17. **Model with mathematics.** An electric utility charges residential customers a \$6 monthly fee plus \$0.04 per kilowatt hour (kWh) for the first 500 kWh and \$0.08/kWh for usage over 500 kWh.
- a. Write a piecewise function $f(x)$ that can be used to determine a customer's monthly bill for using x kWh of electricity.
 - b. Graph the piecewise function.
 - c. A customer uses 613 kWh of electricity in one month. How much should the utility charge the customer? Explain how you determined your answer.

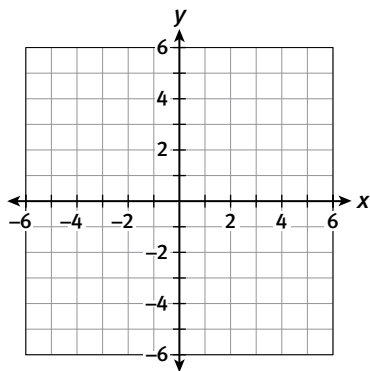
Learning Targets:

- Graph step functions and absolute value functions.
- Describe the attributes of these functions.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Interactive Word Wall, Create Representations, Look for a Pattern, Quickwrite, Think-Pair-Share

A **step function** is a piecewise-defined function whose value remains constant throughout each interval of its domain. Work with your group on Items 1–3. Use your group discussions to clarify the meaning of mathematical concepts and other language used to describe problem information. With your group or your teacher, review background information that will be useful in applying concepts and developing reasonable descriptions and explanations.

1. Graph the step function $f(x) = \begin{cases} -2 & \text{if } x < -3 \\ 1 & \text{if } -3 \leq x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$



2. Describe the graph in Item 1 as completely as possible.

3. **Reason abstractly.** Why do you think the type of function graphed in Item 1 is called a step function?

My Notes

MATH TERMS

A piecewise-defined function with a constant value throughout each interval of its domain is called a **step function**.

DISCUSSION GROUP TIP

Share your description with your group members and list any details you may not have considered before. If you do not know the exact words to describe your ideas, use synonyms or request assistance from group members to help you convey your ideas. Use nonverbal cues such as raising your hand to ask for clarification of others' ideas.

My Notes

TECHNOLOGY TIP

Before graphing a step function on a graphing calculator, go to the Mode window, highlight Dot, and press **ENTER**. Graphing a step function in dot mode will prevent the calculator from connecting breaks in the graph with line segments.

One step function is the greatest integer function, written $f(x) = \lfloor x \rfloor$, which yields a value $f(x)$ that is the greatest integer less than or equal to the value of x . For example, $f(2.7) = \lfloor 2.7 \rfloor = 2$ because the greatest integer less than or equal to 2.7 is 2; and $f(-3.1) = \lfloor -3.1 \rfloor = -4$ because the greatest integer less than or equal to -3.1 is -4 .

4. Graph the greatest integer function on a graphing calculator. To do so, you will need to enter the function as $y = \text{int}(x)$. To locate **int** on the calculator, press **MATH** to reach the Math menu. Then use the right arrow key to access the Number submenu. Finally, select 5: int(.

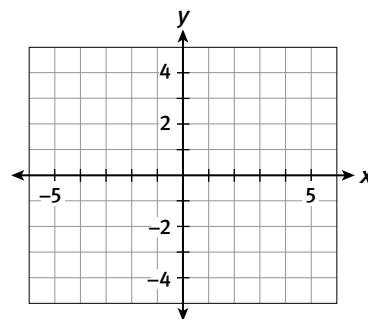
5. **Make sense of problems.** Work with your group. Describe the graph of the greatest integer function as completely as possible. As you listen to the group discussion, take notes to aid comprehension and to help you describe your own ideas to others in your group. Ask questions to clarify ideas and to gain further understanding of key concepts.

Now take a look at a different type of piecewise-defined function.

6. Complete the table and graph the piecewise-defined function.

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



Lesson 4-2

Step Functions and Absolute Value Functions

ACTIVITY 4

continued

7. **Reason quantitatively.** Look back at the graph of $f(x)$ shown in Item 6.
- What are the domain and range of the function?
 - Does the function have a minimum or maximum value? If so, what is it?
 - What are the x -intercept(s) and y -intercept of the function?
 - Describe the symmetry of the graph of the function.

The function $f(x)$ in Item 6 is known as the *absolute value function*. The notation for the function is $f(x) = |x|$. The sharp change in the graph at $x = 0$ is the vertex.

8. Use the piecewise definition of the absolute value function to evaluate each expression.

a. $f(-14) =$

b. $f(8) =$

c. $f(0) =$

d. $f(2 - \sqrt{5}) =$

9. Could you have determined the values of the function in Item 8 another way? Explain.

My Notes

MATH TIP

The absolute value function $f(x) = |x|$ is defined by

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

CONNECT TO AP

The vertex of an absolute value function is an example of a *cusp* in a graph. A graph has a cusp at a point where there is an abrupt change in direction.

My Notes

Check Your Understanding

10. **Construct viable arguments.** Explain why the absolute value function $f(x) = |x|$ is a piecewise-defined function.
11. How is a step function different from other types of piecewise-defined functions?
12. How does the definition of absolute value as a piecewise-defined function relate to the method of solving absolute value equations?

LESSON 4-2 PRACTICE

13. A step function known as the ceiling function, written $g(x) = \lceil x \rceil$, yields the value $g(x)$ that is the least integer greater than or equal to x .
 - a. Graph this step function.
 - b. Find $g(2.4)$, $g(0.13)$, and $g(-8.7)$.

Make sense of problems and persevere in solving them. A day ticket for a ski lift costs \$25 for children at least 6 years old and less than 13 years old. A day ticket for students at least 13 years old and less than 19 years old costs \$45. A day ticket for adults at least 19 years old costs \$60. Use this information for Items 14 and 15.

14. Write the equation of a step function $f(x)$ that can be used to determine the cost in dollars of a day ticket for the ski lift for a person who is x years old.
15. Graph the step function you wrote in Item 14.

Use the absolute value function $h(x) = |x + 2|$ for Items 16–19.

16. Graph the absolute value function.
17. What are the domain and range of the function?
18. What are the coordinates of the vertex of the function's graph?
19. Write the equation for the function using piecewise notation.

My Notes

TECHNOLOGY TIP

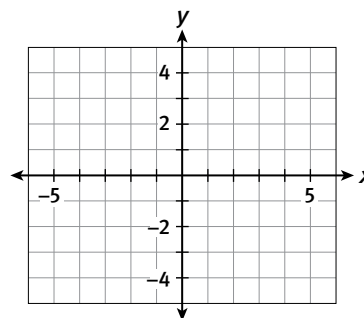
Many graphing calculators use a function called “abs” to represent absolute value.

2. Use the coordinate grid at the right.

a. Graph the parent function $f(x) = |x|$.

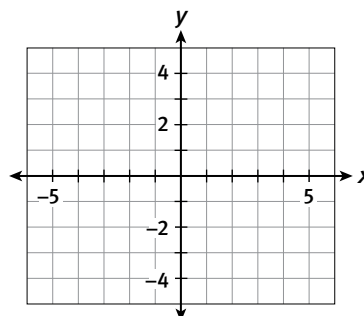
b. Predict the transformation for $g(x) = |x - 3|$ and $h(x) = |-2x|$.

c. Graph the function $g(x) = |x - 3|$ and $h(x) = |-2x|$.



d. What transformations do your graphs show?

3. **Reason abstractly and quantitatively.** Use the results from Item 2 to predict the transformation of $h(x) = |x + 2|$. Then graph the function to confirm or revise your prediction.



The functions in Items 2 and 3 are examples of horizontal translations. A *horizontal translation* occurs when the independent variable, x , is replaced with $x + k$ or with $x - k$.

4. In the absolute value function $f(x) = |x + k|$ with $k > 0$, describe how the graph of the function changes, compared to the parent function.

5. In the absolute value function $f(x) = |x - k|$ with $k > 0$, describe how the graph of the function changes, compared to the parent function.

Lesson 4-3

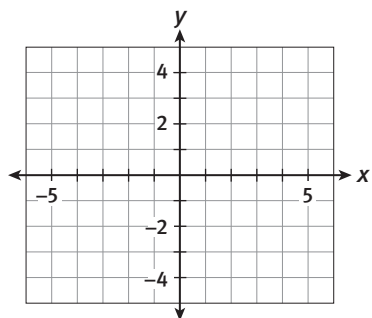
Transforming the Absolute Value Parent Function

ACTIVITY 4

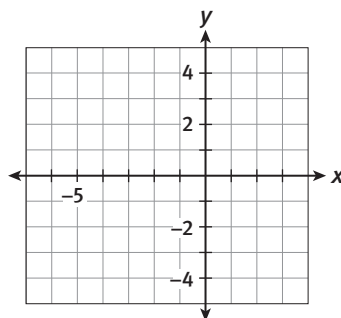
continued

6. Graph each function.

a. $f(x) = |x - 4|$



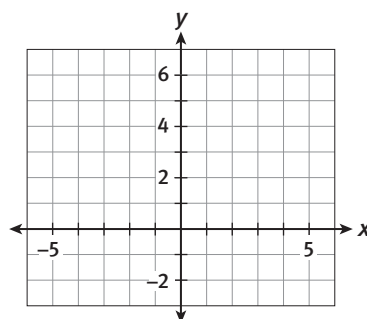
b. $f(x) = |x + 5|$



7. Use the coordinate grid at the right.

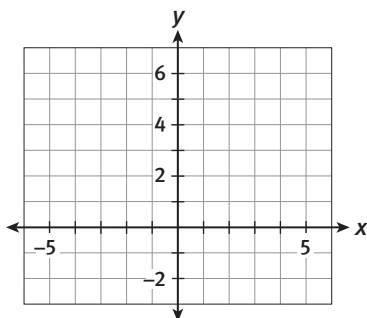
a. Graph the parent function $f(x) = |x|$ and the function $g(x) = |2x|$.

b. Describe the graph of $g(x)$ as a horizontal stretch or horizontal shrink of the graph of the parent function.



8. **Express regularity in repeated reasoning.** Use the results from

Item 7 to predict how the graph of $h(x) = \left| \frac{1}{2}x \right|$ is transformed from the graph of the parent function. Then graph $h(x)$ to confirm or revise your prediction.



My Notes

MATH TIP

A horizontal stretch or shrink by a factor of k maps a point (x, y) on the graph of the original function to the point (kx, y) on the graph of the transformed function.

Similarly, a vertical stretch or shrink by a factor of k maps a point (x, y) on the graph of the original function to the point (x, ky) on the graph of the transformed function.

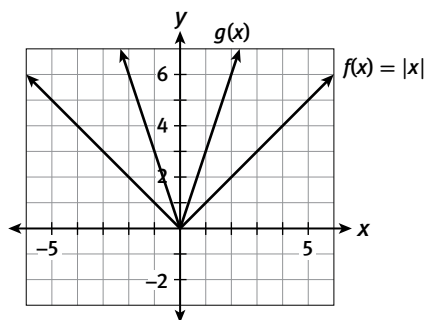
My Notes

9. In the absolute value function $f(x) = |kx|$ with $k > 1$, describe how the graph of the function changes compared to the graph of the parent function. What if $k < -1$?

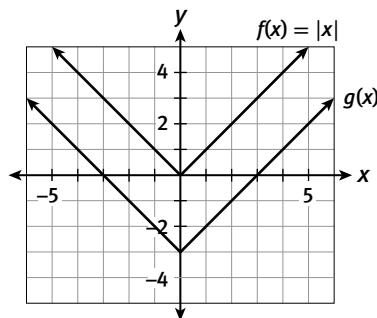
10. In the absolute value function $f(x) = |kx|$ with $0 < k < 1$, describe how the graph of the function changes compared to the graph of the parent function. What if $-1 < k < 0$?

11. Each graph shows a transformation $g(x)$ of the parent function $f(x) = |x|$. Describe the transformation and write the equation of $g(x)$.

a.



b.



Lesson 4-3

Transforming the Absolute Value Parent Function

ACTIVITY 4

continued

Example A

Describe the transformations of $g(x) = 2|x + 3|$ from the parent absolute value function and use them to graph $g(x)$.

Step 1: Describe the transformations.

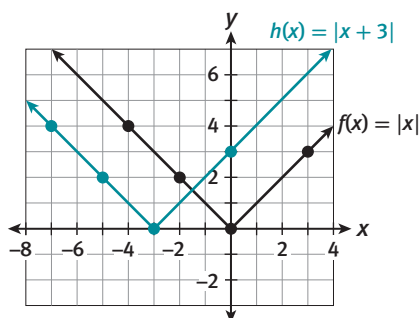
$g(x)$ is a horizontal translation of $f(x) = |x|$ by 3 units to the left, followed by a vertical stretch by a factor of 2.

Apply the horizontal translation first, and then apply the vertical stretch.

Step 2: Apply the horizontal translation.

Graph $f(x) = |x|$. Then shift each point on the graph of $f(x)$ by 3 units to the left. To do so, subtract 3 from the x -coordinates and keep the y -coordinates the same.

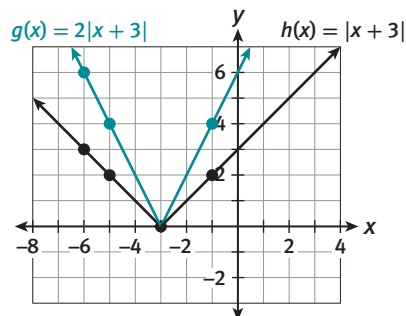
Name the new function $h(x)$. Its equation is $h(x) = |x + 3|$.



Step 3: Apply the vertical stretch.

Now stretch each point on the graph of $h(x)$ vertically by a factor of 2. To do so, keep the x -coordinates the same and multiply the y -coordinates by 2.

Solution: The new function is $g(x) = 2|x + 3|$.



Try These A

For each absolute value function, describe the transformations represented in the rule and use them to graph the function.

a. $h(x) = -|x - 1| + 2$

b. $k(x) = 4|x + 1| - 3$

My Notes

MATH TIP

To graph an absolute value function of the form $g(x) = a|b(x - c)| + d$, apply the transformations of $f(x) = |x|$ in this order:

1. horizontal translation
2. reflection in the y -axis and/or horizontal shrink or stretch
3. reflection in the x -axis and/or vertical shrink or stretch
4. vertical translation

TECHNOLOGY TIP

You can check that you have graphed $g(x)$ correctly by graphing it on a graphing calculator.

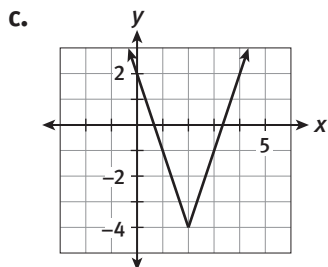
My Notes

Check Your Understanding

12. Graph the function $g(x) = |-x|$. What is the relationship between $g(x)$ and $f(x) = |x|$? Why does this relationship make sense?
13. Compare and contrast a vertical stretch by a factor of 4 with a horizontal stretch by a factor of 4.
14. Without graphing the function, determine the coordinates of the vertex of $f(x) = |x + 2| - 5$. Explain how you determined your answer.

LESSON 4-3 PRACTICE

15. The graph of $g(x)$ is the graph of $f(x) = |x|$ translated 6 units to the right. Write the equation of $g(x)$.
16. Describe the graph of $h(x) = -5|x|$ as one or more transformations of the graph of $f(x) = |x|$.
17. What are the domain and range of $f(x) = |x + 4| - 1$? Explain.
18. Graph each transformation of $f(x) = |x|$.
 - a. $g(x) = |x - 4| + 2$
 - b. $g(x) = |2x| - 3$
 - c. $g(x) = -|x + 4| + 3$
 - d. $g(x) = -3|x + 2| + 4$
19. **Attend to precision.** Write the equation for each transformation of $f(x) = |x|$ described below.
 - a. Translate left 9 units, stretch vertically by a factor of 5, and translate down 23 units.
 - b. Translate left 12 units, stretch horizontally by a factor of 4, and reflect over the x -axis.



ACTIVITY 4 PRACTICE

Write your answers on notebook paper.
Show your work.

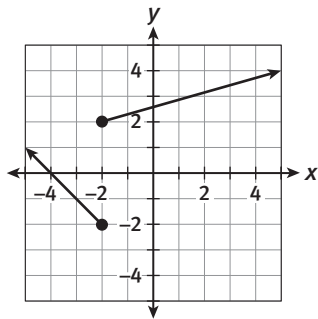
Lesson 4-1

1. Graph each of the following piecewise-defined functions. Then write its domain and range using inequalities, interval notation, and set notation.

a.
$$f(x) = \begin{cases} -3x - 4 & \text{if } x < -1 \\ x & \text{if } x \geq -1 \end{cases}$$

b.
$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ -2x + 3 & \text{if } x > 1 \end{cases}$$

2. Explain why the graph shown below does not represent a function.



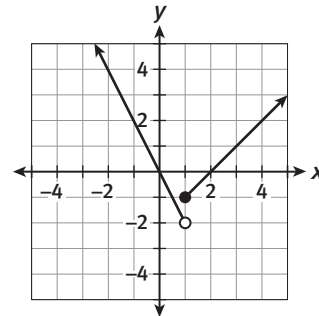
A welder earns \$20 per hour for the first 40 hours she works in a week and \$30 per hour for each hour over 40 hours. Use this information for Items 3–5.

3. Write a piecewise function $f(x)$ that can be used to determine the welder's earnings when she works x hours in a week.
4. Graph the piecewise function.
5. How much does the welder earn when she works 48 hours in a week?
A. \$990 **B.** \$1040
C. \$1200 **D.** \$1440
6. The domain of a function is all real numbers greater than -2 and less than or equal to 8 . Write the domain using an inequality, interval notation, and set notation.
7. The range of a function is $[4, \infty)$. Write the range using an inequality and set notation.

8. Evaluate $f(x)$ for $x = -4$, $x = 1$, and $x = 4$.

$$f(x) = \begin{cases} -5x & \text{if } x < -3 \\ x^2 & \text{if } -3 \leq x < 4 \\ 2x + 4 & \text{if } x \geq 4 \end{cases}$$

9. Write the equation of the piecewise function $f(x)$ shown below.



Lesson 4-2

10. a. Graph the step

$$\text{function } f(x) = \begin{cases} 4 & \text{if } x < -2 \\ 1 & \text{if } -2 \leq x < 3 \\ -3 & \text{if } x \geq 3 \end{cases}$$

- b. What are the domain and range of the step function?
11. It costs \$30 per day or \$90 per week to rent a wallpaper steamer. If the time in days is not a whole number, it is rounded up to the next-greatest day. Customers are given the weekly rate if it is cheaper than using the daily rate.
- a. Write the equation of a step function $f(x)$ that can be used to determine the cost in dollars of renting a wallpaper steamer for x days. Use a domain of $0 < x \leq 7$.
- b. Graph the step function.

ACTIVITY 4

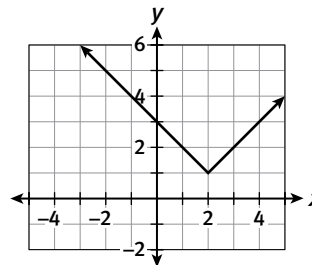
continued

Piecewise-Defined Functions

Absolutely Piece-ful

12. A step function called the *integer part function* gives the value $f(x)$ that is the integer part of x .
- Graph the integer part function.
 - Find $f(-2.1)$, $f(0.5)$, and $f(3.6)$.
13. A step function called the *nearest integer function* gives the value $g(x)$ that is the integer closest to x . For half integers, such as 1.5, 2.5, and 3.5, the nearest integer function gives the value of $g(x)$ that is the even integer closest to x .
- Graph the nearest integer function.
 - Find $g(-2.1)$, $g(0.5)$, and $g(3.6)$.
14. Use the definition of $f(x) = |x|$ to rewrite $f(x) = \frac{|x|}{x}$ as a piecewise-defined function. Then graph the function.
15. Consider the absolute value function $f(x) = |x + 2| - 1$.
- Graph the function.
 - What are the domain and range of the function?
 - What are the x -intercept(s) and y -intercept of the function?
 - Describe the symmetry of the graph.

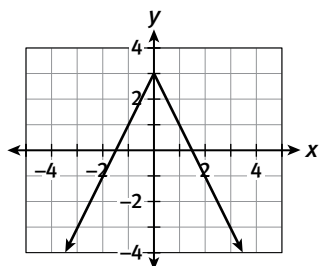
18. Write the equation for each transformation of $f(x) = |x|$ described below.
- translated right 7 units, shrunk vertically by a factor of 0.5, and translated up $\sqrt{5}$ units
 - stretched horizontally by a factor of 5, reflected over the x -axis, and translated down 10 units
 - translated right 9 units and translated down 6 units
19. Which function is shown in the graph?



- $f(x) = |x - 2| + 1$
- $f(x) = |x - 1| + 2$
- $f(x) = |x + 2| + 1$
- $f(x) = |x + 1| + 2$

Lesson 4-3

16. Write the equation of the function $g(x)$ shown in the graph, and describe the graph as a transformation of the graph of $f(x) = |x|$.



17. Graph the following transformations of $f(x) = |x|$. Then identify the transformations.
- $g(x) = |x + 3| - 1$
 - $g(x) = \frac{1}{3}|x| + 2$
 - $g(x) = -2|x - 1| - 1$
 - $g(x) = 5|x - 1| - 4$

MATHEMATICAL PRACTICES

Reason Abstractly and Quantitatively

20. Before answering each part, review them carefully to ensure you understand all the terminology and what is being asked.
- Describe how the graph of $g(x) = |x| + k$ changes compared to the graph of $f(x) = |x|$ when $k > 0$ and when $k < 0$.
 - Describe how the graph of $h(x) = k|x|$ changes compared to the graph of $f(x) = |x|$ when $k > 1$ and when $0 < k < 1$.
 - Describe how the graph of $j(x) = |kx|$ changes compared to the graph of $f(x) = |x|$ when $k < 0$.

Function Composition and Operations

ACTIVITY 5

New from Old

Lesson 5-1 Operations with Functions

Learning Targets:

- Combine functions using arithmetic operations.
- Build functions that model real-world scenarios.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Discussion Groups, Debriefing, Close Reading, Think-Pair-Share, Summarizing, Paraphrasing, Quickwrite

Jim Green has a lawn service called Green's Grass Guaranteed. Tori and Stephan are two of his employees. Tori earns \$10 per hour, and Stephan earns \$8 per hour. Jim sends Tori and Stephan on a job that takes them 4 hours.

- 1. Model with mathematics.** Write a function $t(h)$ to represent Tori's earnings in dollars for working h hours and a function $s(h)$ to represent Stephan's earnings in dollars for working h hours.

- 2.** Find $t(4)$ and $s(4)$ and tell what these values represent in this situation.

- 3.** Find $t(4) + s(4)$ and tell what it represents in this situation.

You can add two functions by adding their function rules.

- 4. a.** Add the functions $t(h)$ and $s(h)$ to find $(t + s)(h)$. Then simplify the function rule.

- b.** What does the function $(t + s)(h)$ represent in this situation?

- 5.** Find $(t + s)(4)$. How does the answer compare to $t(4) + s(4)$?

- 6.** How much will Jim spend on Tori and Stephan's earnings for the 4-hour job?

My Notes

MATH TIP

Addition, subtraction, multiplication, and division are operations on real numbers. You can also perform these operations with functions.

WRITING MATH

The notation $(f + g)(x)$ represents the sum of the functions $f(x)$ and $g(x)$. In other words, $(f + g)(x) = f(x) + g(x)$.

My Notes

7. How much would Jim spend on Tori and Stephan's earnings for a job that takes 6 hours? Explain how you determined your answer.

For a basic tree-trimming job, Jim charges customers a fixed \$25 fee plus \$150 per tree. One of Jim's competitors, Vista Lawn & Garden, charges customers a fixed fee of \$75 plus \$175 per tree for the same service.

8. Write a function $j(t)$ to represent the total charge in dollars for trimming t trees by Jim's company and a function $v(t)$ to represent the total charge in dollars for trimming t trees by Vista.

MATH TIP

When subtracting an algebraic expression, remember to subtract *each* term of the expression. For example, subtract $6x - 2$ from $10x$ as follows.

$$\begin{aligned} 10x - (6x + 2) &= 10x - 6x - 2 \\ &= 4x - 2 \end{aligned}$$

WRITING MATH

The notation $(f - g)(x)$ represents the difference of the functions $f(x)$ and $g(x)$. In other words, $(f - g)(x) = f(x) - g(x)$.

9. a. Subtract $j(t)$ from $v(t)$ to find $(v - j)(t)$. Then simplify the function rule.

- b. What does the function $(v - j)(t)$ represent in this situation?

10. Find $(v - j)(5)$. What does this value represent in this situation?

11. How much will a customer save by choosing Jim's company to trim 8 trees rather than choosing Vista? Explain how you determined your answer.

12. **Look for and make use of structure.** Given $f(x) = 3x + 2$, $g(x) = 2x - 1$, and $h(x) = x^2 - 2x + 8$, find each function and simplify the function rule.

a. $(f + g)(x)$

c. $(h + f)(x)$

e. $(g - f)(x)$

b. $(g + h)(x)$

d. $(f - g)(x)$

f. $(h - g)(x)$

Lesson 5-1

Operations with Functions

ACTIVITY 5

continued

Jim has been asked to make a bid for installing the shrubs around a new office building. In the bid, he needs to include the number of shrubs he can install in an 8-hour day, the cost per shrub including installation, and the total cost of his services for an 8-hour day.

13. a. Write a function $n(h)$ to represent the number of shrubs Jim can install in an 8-hour day when it takes him h hours to install one shrub.
- b. What are the restrictions on the domain of $n(h)$? Explain.
14. Jim will charge \$16 for each shrub. He will also charge \$65 per hour for installation services. Write a function $c(h)$ to represent the amount Jim will charge for a shrub that takes h hours to install.

The total cost of Jim's services for an 8-hour day is equal to the number of shrubs he can install times the charge for each shrub.

15. a. Find the total cost of Jim's services using the functions $n(h)$ and $c(h)$ to find $(n \cdot c)(h)$. Then simplify the function rule.
- b. **Attend to precision.** What are the restrictions on the domain of $(n \cdot c)(h)$?

My Notes

MATH TIP

When considering restrictions on the domain of a real-world function, consider both values of the domain for which the function would be undefined and values of the domain that would not make sense in the situation.

WRITING MATH

The notation $(f \cdot g)(x)$ represents the product of the functions $f(x)$ and $g(x)$. In other words, $(f \cdot g)(x) = f(x) \cdot g(x)$.

My Notes

CONNECT TO BUSINESS

When a company makes a bid on a job, the company states the price at which it is willing to do the job. The company must make its bid high enough to cover all of its expenses. If it bids too high, however, the job may be offered to one of its competitors.

- 16. Reason quantitatively.** Jim estimates that it will take 0.5 hour to install each shrub. Use the functions $n(h)$, $c(h)$, and $(n \cdot c)(h)$ to determine the following values for Jim's bid, and explain how you determined your answers.
- the number of shrubs Jim can install in an 8-hour day
 - the cost per shrub, including installation
 - the total cost of Jim's services for an 8-hour day
- 17.** Explain how you could check your answer to Item 16c.

Jim offers two lawn improvement services, as described in the table.

Lawn Improvement Services

Service	Hourly Charge (\$)	Material Cost for Average Yard (\$)
Compost	40	140
Fertilizer	30	30

- 18. a.** Write a function $c(h)$ to represent the total charge for applying compost to a lawn, where h is the number of hours the job takes.
- b.** Write a function $f(h)$ to represent the total charge for applying fertilizer to a lawn, where h is the number of hours the job takes.

Lesson 5-1

Operations with Functions

ACTIVITY 5

continued

19. a. Divide $c(h)$ by $f(h)$ to find $(c \div f)(h)$ given that $f(h) \neq 0$.
- b. What does the function $(c \div f)(h)$ represent in this situation?
20. Find $(c \div f)(4)$. What does this value represent in this situation?
21. **Look for and make use of structure.** Given $f(x) = 2x$, $g(x) = x + 3$, and $h(x) = 2x + 6$, find each function and simplify the function rule. Note any values that must be excluded from the domain.
- a. $(f \cdot g)(x)$
- b. $(g \cdot h)(x)$
- c. $(f \div h)(x)$, $h(x) \neq 0$
- d. $(h \div g)(x)$, $g(x) \neq 0$
- e. $(g \div f)(x)$

My Notes

WRITING MATH

The notation $(f \div g)(x)$, $g(x) \neq 0$ represents the quotient of the functions $f(x)$ and $g(x)$ given that $g(x) \neq 0$. In other words, $(f \div g)(x) = f(x) \div g(x)$, $g(x) \neq 0$.

MATH TIP

You may be able to simplify the function rules in Items 21c, d, and e by factoring the expression's numerator and denominator and dividing out common factors.

My Notes

DISCUSSION GROUP TIP

As you listen to your group members' discussions, you may hear math terms or other words that you do not know. Use your math notebook to record words that are frequently used. Ask for clarification of their meaning, and make notes to help you remember and use those words in your own communications.

22. Discuss and then answer this question with your group. How are operations on functions similar to and different from operations on real numbers?

Check Your Understanding

23. Given that $f(x) = 2x + 1$ and $g(x) = 3x - 2$, what value(s) of x are excluded from the domain of $(f \div g)(x)$? Explain your answer.
24. Make a conjecture about whether addition of functions is commutative. Give an example that supports your conjecture.
25. Given that $h(x) = 4x + 5$ and $(h - j)(x) = x - 2$, find $j(x)$. Explain how you determined your answer.

LESSON 5-1 PRACTICE

For Items 26–30, use the following functions.

$$f(x) = 5x + 1$$

$$g(x) = 3x - 4$$

Find each function and simplify the function rule. Note any values that must be excluded from the domain.

26. $(f + g)(x)$

27. $(f - g)(x)$

28. $(f \cdot g)(x)$

29. $(f \div g)(x), g(x) \neq 0$

30. A student incorrectly found $(g - f)(x)$ as follows. What mistake did the student make, and what is the correct answer?

$$(g - f)(x) = 3x - 4 - 5x + 1 = -2x - 3$$

31. **Make sense of problems and persevere in solving them.** Jim plans to make a radio ad for his lawn company. The function $a(t) = 800 + 84t$ gives the cost of making the ad and running it t times on an AM station. The function $f(t) = 264t$ gives the cost of running the ad t times on a more popular FM station.

- a. Find $(a + f)(t)$ and tell what it represents in this situation.
- b. Find $(a + f)(12)$ and tell what it represents in this situation.

My Notes

MATH TIP

In a linear function $f(x) = mx + b$, the y -intercept is b . The variable m is the rate of change in the values of the function—the change of units of $f(x)$ per change of unit of x . When the function is graphed, the rate of change is interpreted as the slope. So $y = mx + b$ is called the slope-intercept form of a linear equation.

The functions in Items 2 and 4 relate three quantities that vary, based on the needs of Jim’s customers:

- The size in acres a of the property
- The time in hours t needed to perform the work
- The cost in dollars c of doing the work.

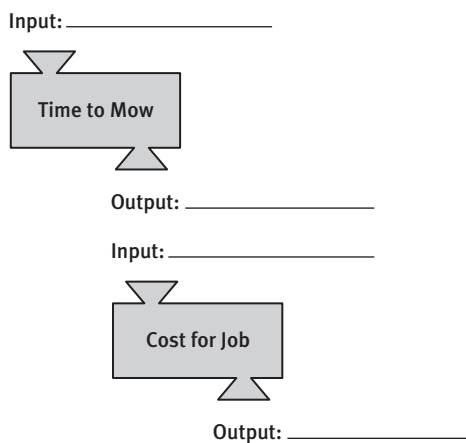
6. Attend to precision. Complete the table below by writing the rate of change with units and finding the slope of the graph of the function.

Function	Rate of Change (with units)	Slope
$c(t) =$		
$t(a) =$		

7. Complete the table below by naming the measurement units for the domain and range of each function.

Function Notation	Description of Function	Domain (units)	Range (units)
$c(t)$	cost for job		
$t(a)$	time to mow		

8. Calculating the cost to mow a lawn is a two-step process. Complete the graphic organizer below by describing the input and output, including units, for each part of the process.



Lesson 5-2

Function Composition

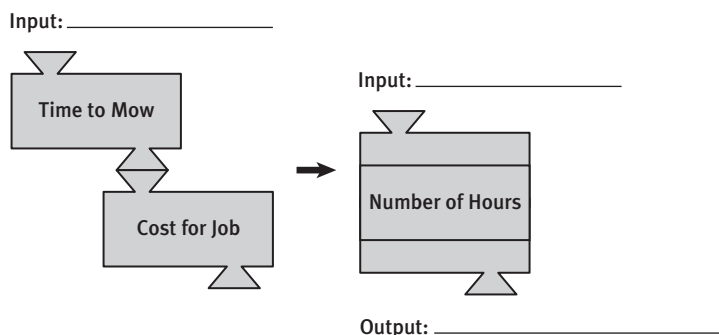
ACTIVITY 5

continued

The graphic organizer shows an operation on two functions, called a **composition**. The function that results from using the output of the first function as the input for the second function is a **composite function**.

In this context, the composite function is formed by the time-to-mow function and the cost-for-job function. Its domain is the input for the time function, and its range is the output from the cost function.

- 9. Make sense of problems.** The cost to mow is a composite function. Describe its input and output as you did in Item 8.



When a composite function is formed, the function is often named to show the functions used to create it. The cost-to-mow function, $c(t(a))$, is composed of the cost-for-job and the time-to-mow functions.

The $c(t(a))$ notation implies that a was assigned a value $t(a)$ by the time-to-mow function. Then the resulting $t(a)$ value was assigned a value $c(t(a))$ by the cost-to-mow function.

- 10.** Complete the table by writing a description for the composite function $c(t(a))$. Then name the measurement units of the domain and range.

Function Notation	Description of Function	
$c(t(a))$		
	Domain (units)	Range (units)

My Notes

MATH TERMS

A **composition** is an operation on two functions that forms a new function. To form the new function, the rule for the first function is used as the input for the second function.

A **composite function** is the function that results from the composition of two functions. The range of the first function becomes the domain for the second function.

My Notes

Jim wants to write one cost function for mowing a acres of property. To write the cost c as a function of a acres of property, he substitutes $t(a)$ into the cost function and simplifies.

$$\begin{aligned} c(t) &= c(t(a)) && \text{Substitute } t(a) \text{ for } t \text{ in the cost function.} \\ c(t(a)) &= c(4a) && t(a) = 4a, \text{ so write the function in terms of } a. \\ &= 30 + 20(4a) && \text{Substitute } 4a \text{ for } t \text{ in the original } c(t) \text{ function.} \\ c(t(a)) &= 30 + 80a \end{aligned}$$

11. **Attend to precision.** Write a sentence to explain what the expression $c(t(2))$ represents. Include appropriate units in your explanation.

12. **Construct viable arguments.** Why might Jim want a single function to determine the cost of a job when he knows the total number of acres?

13. Explain what the expression $c(t(50))$ represents. Include appropriate units in your explanation.

14. Explain what information the equation $c(t(a)) = 50$ represents. Include appropriate units in your explanation.

My Notes

Learning Targets:

- Write the composition of two functions.
- Evaluate the composition of two functions.

SUGGESTED LEARNING STRATEGIES: Note Taking, Create Representations, Think-Pair-Share, Group Presentation, Debriefing

A *composition of functions* forms a new function by substituting the output of the inner function into the outer function. The function $y = f(g(x))$ is a composition of f and g where g is the inner function and f is the outer function.

1. The tables show information about Jim's mowing service. Use the tables to evaluate each expression. Then tell what the expression represents.

Area of Property a (acres)	Time to Mow $t(a)$ (hours)
1	4
2	8
3	12
4	16

Time to Mow t (hours)	Cost to Mow $c(t)$ (\$)
4	110
8	190
12	270
16	350

- a. $t(4)$
- b. $c(4)$
- c. $c(t(1))$
- d. $c(t(4))$

2. **Reason quantitatively.** Use the tables of values below to evaluate each expression.

x	$f(x)$
1	3
2	2
3	1
4	4

x	$g(x)$
1	4
2	3
3	2
4	1

- a. $f(3)$
- b. $g(3)$
- c. $g(f(3))$
- d. $f(g(3))$

MATH TIP

The order matters when you compose two functions. $y = g(f(x))$ and $y = f(g(x))$ are two different functions.

Lesson 5-3

More Function Composition

ACTIVITY 5

continued

3. Using f and g from Item 2, complete each table of values to represent the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

a.

x	$(f \circ g)(x) = f(g(x))$
1	
2	
3	
4	

b.

x	$(g \circ f)(x) = g(f(x))$
1	
2	
3	
4	

Check Your Understanding

4. What does the notation $(g \circ h)(t)$ represent? What is another way you can write $(g \circ h)(t)$?
5. **Reason abstractly.** Explain how $(f \circ g)(x)$ is different from $(f \cdot g)(x)$.
6. Given that $p(t) = t^2 + 4$ and $q(t) = t + 3$, write the equation for $(p \circ q)(t)$. Explain how you determined your answer.

For Items 7–11, use these three functions:

- $f(x) = x^2$
- $g(x) = 2x - 1$
- $h(x) = 4x - 3$

7. Evaluate each expression.

a. $g(f(2))$ b. $f(g(2))$

8. Write each composite function in terms of x .

a. $y = g(f(x))$ b. $y = f(g(x))$

9. Verify that you composed g and f correctly by evaluating $g(f(2))$ and $f(g(2))$ using the functions you wrote in Item 8. Compare your answers with those from Item 7.

My Notes

WRITING MATH

The notation $(f \circ g)(x)$ represents a composition of two functions.

$$(f \circ g)(x) = f(g(x))$$

Read the notation as “ f of g of x .”

CONNECT TO AP

In AP Calculus, you will identify the “inner” function and the “outer” function that form a composite function.

For example, the function $h(x) = f(g(x)) = (2x + 3)^2$ could be the composition of the inner function $g(x) = 2x + 3$ and the outer function $f(x) = x^2$.

My Notes

MATH TIP

As shown in Item 11, the inner and outer functions that form a composite function can be the same function.

10. a. Evaluate $h(g(3))$.

b. Write the composition $(h \circ g)(x)$ in terms of x .

11. a. Evaluate $g(g(2))$.

b. Write the composition $(g \circ g)(x)$ in terms of x .

Check Your Understanding

12. Explain how you found the rule for the composition $(g \circ g)(x)$ in Item 11b.

13. Given that $p(n) = 4n$ and $q(n) = n + 2$, for what value of n is $(p \circ q)(n) = 8$? Explain how you determined your answer.

LESSON 5-3 PRACTICE

For Items 14 and 15, use the following functions:

- $f(x) = 5x + 1$
- $g(x) = 3x - 4$

14. Evaluate $f(2)$, $g(2)$, $(f \circ g)(2)$, and $(g \circ f)(2)$.

15. Write the composite functions $h(x) = g(f(x))$ and $k(x) = f(g(x))$.

The jeans at a store are on sale for 20% off, and the sales tax rate is 8%. Use this information for Items 16–18.

16. Write a function $s(p)$ that gives the sale price of a pair of jeans regularly priced at p dollars.

17. Write a function $t(p)$ that gives the total cost including tax for a pair of jeans priced at p dollars.

18. **Construct viable arguments.** A customer wants to buy a pair of jeans regularly priced at \$25. Does it matter whether the sales clerk applies the sale discount first or adds on the sales tax first to find the total cost? Use compositions of the functions s and t to support your answer.

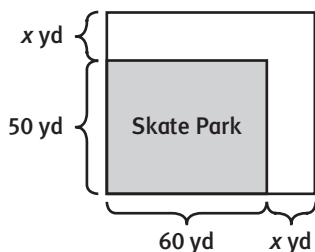
ACTIVITY 5 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 5-1

Use $f(x) = 5x + 2$, $g(x) = 3 - x$, and $h(x) = x - 3$ to answer Items 1–8. Find each function and simplify the function rule. Note any values that must be excluded from the domain.

1. $(f + g)(x)$
2. $(h + g)(x)$
3. $(f - g)(x)$
4. $(h - f)(x)$
5. $(f \cdot g)(x)$
6. $(g \cdot h)(x)$
7. $(f \div g)(x)$, $g(x) \neq 0$
8. $(g \div h)(x)$, $h(x) \neq 0$
9. A rectangular skate park is 60 yards long and 50 yards wide. Plans call for increasing both the length and the width of the park by x yards.



- a. Write a function $l(x)$ that gives the new length of the skate park in terms of x .
 - b. Write a function $w(x)$ that gives the new width of the skate park in terms of x .
 - c. What does $(l \cdot w)(x)$ represent in this situation? Write and simplify the equation for $(l \cdot w)(x)$.
 - d. Find $(l \cdot w)(5)$, and tell what it represents in this situation.
10. Given that $p(n) = 4n^2 + 4n - 6$ and $q(n) = n^2 - 5n + 8$, find $(p - q)(3)$.
- A. 26
 - B. 38
 - C. 40
 - D. 42

11. Make a conjecture about whether subtraction of functions is commutative. Give an example that supports your answer.
12. The cost in dollars of renting a car for d days is given by $c(d) = 22d + 25$. The cost in dollars of renting a hotel room for d days is given by $h(d) = 74d$.
 - a. What does $(c + h)(d)$ represent in this situation? Write and simplify the equation for $(c + h)(d)$.
 - b. For what value of d is $(c + h)(d) = 600$? What does this value of d represent in this situation?

Lesson 5-2

Jim wants to calculate the cost of running his lawn mowers. The mowers consume 2.5 gallons of gasoline each hour. Gasoline costs \$3.50 per gallon.

13. Write a function $g(h)$ that gives the number of gallons g that the mowers will use in h hours. Identify the units of the domain and range.
14. Write a function $c(g)$ for the cost c in dollars for g gallons of gasoline. Identify the units of the domain and range.
15. Use composition of functions to create a function for the cost c in dollars of gasoline to mow h hours. Identify the units of the domain and range. Then explain how the domain and range of the composite function are related to the domain and range of $g(h)$ and $c(g)$.
16. Use the composite function in Item 15 to determine the cost of gasoline to mow 12 hours. Show your work.
17. What is the slope of the composite function, and what does it represent in this situation?

An empty swimming pool is shaped like a rectangular prism with a length of 18 feet and a width of 9 feet. Once water begins to be pumped into the pool, the depth of the water increases at a rate of 0.5 foot per hour.

18. Write a function $d(t)$ that gives the depth in feet of the water in the pool after t hours.
 19. Write a function $v(d)$ that gives the volume in cubic feet of the water in the pool when the depth of the water is d feet.
 20. Write the equation of the composite function $v(d(t))$, and tell what this function represents in this situation.
 21. What is $v(d(4))$, and what does it represent in this situation?
 22. The range of the function $d(t)$ is $0 \leq d \leq 4$. Based on this information, what is the greatest volume of water the pool can hold?
31. Given that $(r \circ s)(t) = 2t + 11$, which could be the functions r and s ?
 - A. $r(t) = t + 1, s(t) = 2t + 5$
 - B. $r(t) = t + 5, s(t) = 2t + 1$
 - C. $r(t) = 2t + 1, s(t) = t + 5$
 - D. $r(t) = 2t + 5, s(t) = t + 1$
 32. What is the composition $f \circ g$ if $f(x) = 4 - 2x$ and $g(x) = 3x^2$?
 - A. $f(g(x)) = 12x^2 - 6x^3$
 - B. $f(g(x)) = 4 - 6x^2$
 - C. $f(g(x)) = 3(4 - 2x)^2$
 - D. $f(g(x)) = 12 - 12x^4$

Use $f(x) = 5x + 2$ and $g(x) = 3 - x$ to answer Items 33–35.

33. What is the value of $f(g(-1))$ and $g(f(-1))$?
34. What is the composite function $y = f(g(x))$?
35. What is the composite function $y = g(f(x))$?

Lesson 5-3

Use $f(x) = x^2$, $g(x) = x + 5$, and $h(x) = 4x - 6$ to answer Items 23–28. Find each function and simplify the function rule.

23. $(f \circ g)(x)$
24. $(g \circ f)(x)$
25. $(f \circ h)(x)$
26. $(h \circ f)(x)$
27. $(g \circ h)(x)$
28. $(h \circ g)(x)$

The function $c(f) = \frac{5}{9}(f - 32)$ converts a temperature f in degrees Fahrenheit to degrees Celsius. The function $k(c) = c + 273$ converts a temperature c in degrees Celsius to units called kelvins.

29. Write a composite function that can be used to convert a temperature in degrees Fahrenheit to kelvins.
30. In Item 29, does it matter whether you wrote $(c \circ k)(f)$ or $(k \circ c)(f)$? Explain.

MATHEMATICAL PRACTICES

Model with Mathematics

36. A store is discounting all of its television sets by \$50 for an after-Thanksgiving sale. The sales tax rate is 7.5%.
 - a. Write a function $s(p)$ that gives the sale price of a television regularly priced at p dollars.
 - b. Write a function $t(p)$ that gives the total cost including tax for a television priced at p dollars.
 - c. A customer wants to buy a television regularly priced at \$800. Does it matter whether the sales clerk applies the sale discount first or adds on the sales tax first to find the total cost? Use compositions of the functions s and t to support your answer.

Inverse Functions

Old from New

Lesson 6-1 Finding Inverse Functions

Learning Targets:

- Find the inverse of a function.
- Write the inverse using the proper notation.

SUGGESTED LEARNING STRATEGIES: Questioning the Text, Think-Pair-Share, Work Backward, Debriefing, Quickwrite, Create Representations, Look for a Pattern, Group Presentation, Note Taking

Green's Grass Guaranteed charges businesses a flat fee of \$30 plus \$80 per acre for lawn mowing. For residential customers who may have a more limited budget, Jim Green needs to determine the size of the yard he could mow for a particular weekly fee.

Work on Items 1–10 with your group.

1. The cost function F is $C = F(A)$. It can be written $C = 30 + 80A$, where C is the cost to mow A acres. Use the function to determine what part of an acre Jim could mow for each weekly fee.
 - a. \$60
 - b. \$80
 - c. \$110

To make a profit and still charge a fair price, Jim needs a function for calculating the maximum acreage that he can mow, based on the amount of money a customer is willing to spend.

2. **Attend to precision.** What are the units of the domain and range of the cost function in Item 1?
3. **Make use of structure.** Solve the function equation from Item 1 for A in terms of C .
$$C = 30 + 80A$$
4. Write the answer equation from Item 3 using function notation, where G is the acreage function.

My Notes

DISCUSSION GROUP TIP

As you work in groups, review the problem scenario carefully and explore together the information provided and how to use it to create a potential solution. Discuss your understanding of the problem and ask peers or your teacher to clarify any areas that are not clear.

My Notes

DISCUSSION GROUP TIPS

As you share ideas in your group, ask your group members or your teacher for clarification of any language, terms, or concepts that you do not understand.

As you prepare your presentation, remember to use words that will help your classmates understand the problem. Also, be careful to communicate mathematical terms correctly to describe the application of mathematical concepts and potential solutions.

5. What are the units of the domain and range of the function G ?

6. **Reason abstractly.** Discuss the following question with your group and prepare to share your group's answer with the rest of the class. How are the domain and range of $F(A)$ related to those of $G(C)$? Use your response to Item 3 to explain why the relationship exists.

7. Use the appropriate functions to evaluate each expression.
 - a. $G(60)$

 - b. $F(G(60))$

 - c. $F(2)$

 - d. $G(F(2))$

8. **Attend to precision.** Interpret the meaning of each expression and its corresponding value in Item 7. Be sure to include units in your explanation.
 - a.

 - b.

 - c.

 - d.

Lesson 6-1

Finding Inverse Functions

ACTIVITY 6

continued

9. In general, what is the result when you evaluate $F(G(x))$ and $G(F(x))$?

10. What do the answers in Items 7–9 suggest about F and G ?

Two functions f and g are **inverse functions** if and only if:

$$\begin{aligned} f(g(x)) &= x \text{ for all } x \text{ in the domain of } g, \\ &\text{and} \\ g(f(x)) &= x \text{ for all } x \text{ in the domain of } f. \end{aligned}$$

The function notation f^{-1} denotes the inverse of function f and is read “ f inverse.”

Item 6 showed that the domain of a function is the range of its inverse. Likewise, the range of a function is the domain of its inverse. To find the inverse of a function algebraically, interchange the x and y variables and then solve for y .

Example A

Find the inverse of the function $f(x) = 2x - 4$.

Step 1: Let $y = f(x)$.

$$y = 2x - 4$$

Step 2: Interchange the x and y variables.

$$x = 2y - 4$$

Step 3: Solve for y .

$$x + 4 = 2y$$

$$y = \frac{x + 4}{2}$$

Step 4: Let $y = f^{-1}(x)$.

$$f^{-1}(x) = \frac{x + 4}{2}$$

Solution: $f^{-1}(x) = \frac{x + 4}{2}$

Try These A

Find the inverse of each function.

a. $f(x) = -3x + 8$

b. $g(x) = \frac{1}{4}(x + 12)$

c. $h(x) = \frac{2}{3}x - 5$

d. $j(x) = \frac{3x - 2}{6}$

My Notes

MATH TERMS

Functions f and g are **inverse functions** if and only if $f(g(x)) = x$ for all x in the domain of g and $g(f(x)) = x$ for all x in the domain of f .

WRITING MATH

If f and g are inverse functions, you can also write two equivalent composite functions:

$$\begin{aligned} f \circ g &= x \\ g \circ f &= x \end{aligned}$$

MATH TIP

The -1 superscript in the function notation f^{-1} is not an exponent, and $f^{-1} \neq \frac{1}{f}$ when referring to functions.

However, for any number n , the expression n^{-1} is the multiplicative inverse, or reciprocal, of n .

My Notes

Check Your Understanding

11. Given that f and g are inverse functions, explain how you can find $f(g(20))$ without knowing the equations for f and g .
12. **Critique the reasoning of others.** A student claims that $h(x) = 3x$ and $j(x) = -3x$ are inverse functions. Is the student correct? Justify your answer.
13. The domain of a function is $x \geq 3$, and the range of the function is $y \leq -2$. What are the domain and range of the inverse function? Explain your answer.
14. Mariana's average running speed is 6 miles per hour.
 - a. Write a function $D(T)$ that gives the distance in miles Mariana covers when running for T minutes.
 - b. What are the units of the domain and range of $D(T)$? Write the domain and range in both interval notation and set notation.
 - c. What are the units of the domain and range of the inverse of $D(T)$? Write the domain and range of the inverse function in both interval notation and set notation.
 - d. How many minutes will it take Mariana to run 2.5 miles? Explain how you can find the answer by using the inverse of $D(T)$.

LESSON 6-1 PRACTICE

Look for and make use of structure. The function $T = F(H)$ estimates the temperature (degrees Celsius) on a mountain given the height (in meters) above sea level. Use the function $T = 50 - \frac{H}{20}$.

15. What is $F(500)$? What does $F(500)$ mean?
16. Find H in terms of T . Label this function G .
17. What is $G(25)$? What does $G(25)$ mean?
18. Are the functions F and G inverses of each other? Explain.

Find the inverse of each function.

19. $f(x) = 3x + 6$
20. $g(x) = -\frac{1}{2}x$
21. $h(x) = \frac{x-20}{4}$
22. $j(x) = 5(x-1)$

Learning Targets:

- Use composition of functions to determine if functions are inverses of each other.
- Graph inverse functions and identify the symmetry.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations, Group Presentation, Quickwrite, Debriefing, Discussion Groups, Self Revision/Peer Revision

You can use the definition of inverse functions to show that two functions are inverses of each other.

Example A

Use the definition of inverse functions to **prove** that $f(x) = 2x - 4$ and $f^{-1}(x) = \frac{x+4}{2}$ are inverse functions.

Step 1: Compose f and f^{-1} . $f(f^{-1}(x)) = 2(f^{-1}(x)) - 4$

Step 2: Substitute f^{-1} into f . $= 2\left(\frac{x+4}{2}\right) - 4$

Step 3: Simplify. $= x + 4 - 4$
 $= x$

Step 4: Compose f^{-1} and f . $f^{-1}(f(x)) = \frac{f(x)+4}{2}$

Step 5: Substitute f into f^{-1} . $= \frac{2x - 4 + 4}{2}$

Step 6: Simplify. $= \frac{2x}{2}$
 $= x$

Solution: $f(x) = 2x - 4$ and $f^{-1}(x) = \frac{x+4}{2}$ are inverse functions.

Try These A

Make use of structure. Find the inverse of the function. Then use the definition to prove the functions are inverses. Show your work.

a. $f(x) = 4x - 14$

b. $g(x) = \frac{1}{2}x + 3$

My Notes

ACADEMIC VOCABULARY

When you **prove** a statement, you use logical reasoning to show that it is true.

My Notes

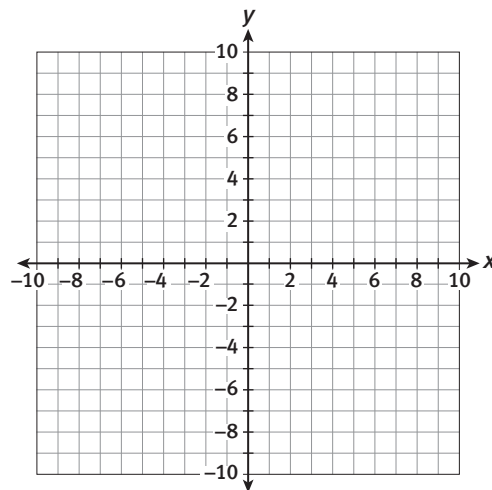
Check Your Understanding

- Suppose that the domain of $f(x) = 2x - 4$ in Example A was restricted to $\{x \mid x \in \mathbb{R}, x \geq 2\}$. What would be the domain and range in set notation of $f^{-1}(x)$? Explain your answer.
- Construct viable arguments.** Explain how to prove that two functions $h(x)$ and $j(x)$ are inverse functions.
- The domain of $p(t)$ is $[0, \infty)$. The range of $q(t)$ is $(-\infty, 0]$. Based on this information, could $q(t)$ be the inverse of $p(t)$? Explain your answer.

You can use the relationship between the domain and range of a function and its inverse to graph the inverse of a function. If (x, y) is a point on the graph of a given function, then (y, x) is a point on the graph of its inverse.

- Complete the table of values for $f(x) = 3x - 2$. Use the values to graph the function on the coordinate axes below.

x	$f(x)$
-2	
0	
3	
4	



- Use the table in Item 4 to make a table of values for the inverse of f . Then graph the inverse on the same coordinate axes.

x	$f^{-1}(x)$

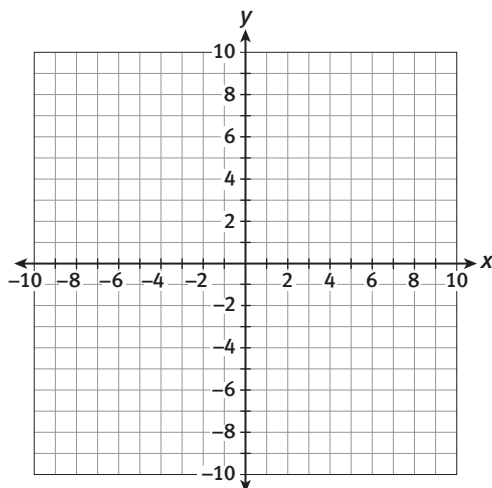
Lesson 6-2

Graphs of Inverse Functions

ACTIVITY 6

continued

- Show the graph of $y = x$ as a dotted line on the coordinate axes in Item 4. Describe any symmetry among the three graphs.
- Find the inverse of $f(x) = x - 4$.
- a. Model with mathematics.** Graph $f(x) = x - 4$, its inverse $f^{-1}(x)$ from Item 7, and the dotted line $y = x$ on the coordinate axes.



- b.** Describe any symmetry that you see on the graph in Item 8a.

Check Your Understanding

- Graph the function $f(x) = 2$ and its inverse on the same coordinate plane. Is the inverse of $f(x) = 2$ a function? Explain your answer.
- What is the relationship between the slope of a nonhorizontal linear function and the slope of its inverse function? Explain your reasoning.
- What is the relationship between the x - and y -intercepts of a function and the x - and y -intercepts of its inverse? Explain your reasoning.

My Notes

CONNECT TO GEOMETRY

Geometric figures in the coordinate plane can have symmetry about a point, a line, or both.

MATH TIP

Recall that the slope of a linear function is equal to $\frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are two points on the function's graph.

My Notes

LESSON 6-2 PRACTICE

In Items 12–14, find the inverse of each function. Use the definition of inverse functions to verify that the two functions are inverses.

12. $f(x) = 6 - 3x$

13. $g(x) = x + 2$

14. $h(x) = -x + 5$

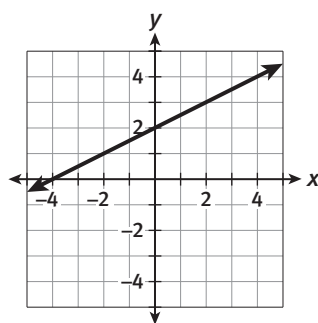
15. **Express regularity in repeated reasoning.** Using your results in Items 12–14, state whether each statement is true or false. Explain your reasoning.

a. A function and its inverse always intersect.

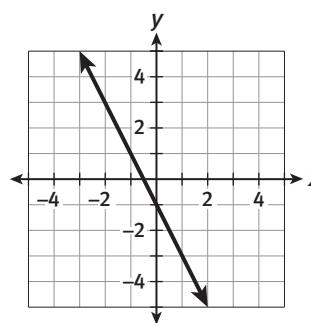
b. The rule for a function cannot equal the rule for its inverse.

In Items 16 and 17, graph the inverse of each function shown on the coordinate plane.

16.



17.



18. **Reason abstractly and quantitatively.** Summarize the relationship between a function and its inverse by listing at least three statements that must be true if two functions are inverses of each other.

ACTIVITY 6 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 6-1

Mark's landscaping business Mowing Madness uses the function $c = F(a)$ to find the cost c of mowing a acres of land. He charges a \$50 fee plus \$60 per acre. Mark's cost-calculating function is $c = 60a + 50$.

Use this function for Items 1–7.

1. What is $F(40)$? What does $F(40)$ mean?
2. Find a in terms of c . Label this function G .
3. What is $G(170)$? What does $G(170)$ mean?
4.
 - a. What are the units of the domain and range of $F(a)$?
 - b. What are the units of the domain and range of $G(c)$?
5.
 - a. What are the domain and range of $F(a)$ in interval notation?
 - b. What are the domain and range of $G(c)$ in interval notation?
6. Are $F(a)$ and $G(c)$ inverse functions? Explain your answer.
7. A customer has \$200 to spend on mowing. How many acres will Mark mow for this amount? Explain how you determined your answer.
8. Find the inverse of each function.
 - a. $f(x) = 2x - 10$
 - b. $g(x) = \frac{x+5}{4}$
 - c. $h(x) = \frac{1}{6}(x-8)$
 - d. $j(x) = -5x + 2$
9. Given that $f(1) = 5$, which of the following statements must be true?

A. $f^{-1}(1) = -5$	B. $f^{-1}(1) = 5$
C. $f^{-1}(5) = -1$	D. $f^{-1}(5) = 1$

The function $m = F(a) = \frac{a}{8}$ gives the distance in inches on a map between two points that are actually a miles apart. Use this function for Items 10–13.

10. What is $F(50)$? What does $F(50)$ represent?
11.
 - a. What is the inverse of $F(a)$? Label this function G , and tell how you determined the rule for the inverse function.
 - b. Tell what the inverse function represents.
12. What is $G(3)$? What does $G(3)$ represent?
13. Two towns on the map are $4\frac{1}{2}$ inches apart. What is the actual distance in miles between the two towns? Explain how you determined your answer.
14. What is the inverse of the function $p(t) = 6t + 8$?
 - A. $p^{-1}(t) = -6t - 8$
 - B. $p^{-1}(t) = \frac{-t+8}{6}$
 - C. $p^{-1}(t) = \frac{t-8}{6}$
 - D. $p^{-1}(t) = \frac{1}{6}t - 8$

Lesson 6-2

15. Use the definition of *inverse* to determine whether or not each pair of functions are inverses.
 - a. $f(x) = 5x - 3$, $g(x) = \frac{x}{5} + 3$
 - b. $f(x) = \frac{x}{2} + 3$, $g(x) = 2x - 6$
 - c. $f(x) = 2(x - 4)$, $g(x) = \frac{1}{2}x + 4$
 - d. $f(x) = x + 3$, $g(x) = -x - 3$

16. Use a graph to determine which two of the three functions listed below are inverses.

a. $f(x) = \frac{2}{3}x + 6$ b. $g(x) = \frac{3}{2}x - 6$

c. $h(x) = \frac{3}{2}x - 9$

17. Write the inverse of the function defined by the table shown below.

x	-2	0	1	3	5
$f(x)$	4	-1	2	0	5

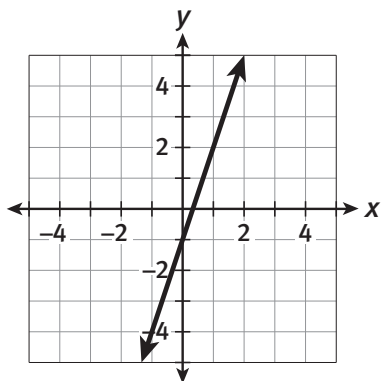
18. Find the inverse of each function. Then use the definition of inverse functions to verify that the two functions are inverses.

a. $f(x) = -3x + 3$

b. $g(x) = 0.25x + 0.6$

19. a. Graph the absolute value function $f(x) = |x| + 2$.
 b. Graph the inverse of $f(x)$ on the same coordinate plane. Explain how you graphed the inverse.
 c. Give the domain and range of $f(x)$ and its inverse using set notation.
 d. Is the inverse of $f(x)$ a function? Explain your answer.

20. Graph the inverse of the function shown below.



21. Graph each function and its inverse on the same coordinate plane.

a. $f(x) = 2x + 4$

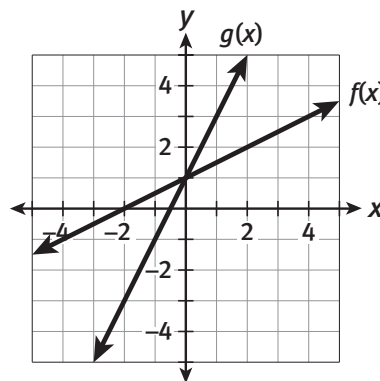
b. $g(x) = -x - 2$

22. The graph of a function passes through the point $(-3, 4)$. Based on this information, which point *must* lie on the graph of the function's inverse?

A. $(-4, 3)$ B. $(-3, 4)$

C. $(3, -4)$ D. $(4, -3)$

23. Explain why the functions $f(x)$ and $g(x)$, graphed below, are not inverse functions.



24. A function $h(x)$ has two different x -intercepts. Is the inverse of $h(x)$ a function? Explain your answer.

25. Describe a method for determining whether a function $f(x)$ is its own inverse.

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

26. A student says that the functions $f(x) = 2x + 2$ and $g(x) = 2x - 2$ are inverse functions because their graphs are parallel. Is the student's reasoning correct? Justify your answer.

CURRENCY CONVERSION

Kathryn and Gaby are enrolled in a university program to study abroad in Spain and then in South Africa. They realize that they will have to convert US dollars (USD) to euros (EUR) in Spain, and then convert EUR into South African rand (ZAR) for their time in South Africa. They identified a currency exchange service in Spain that will convert D dollars to euros with the function $E(D) = 0.64D - 5$ and a currency exchange service in South Africa that will convert E euros to rand using $R(E) = 12.1E - 10$.

Use the information above to solve the following problems. Show your work.

1. For each function, give the units for the domain and range.
2. If Kathryn converts 450 USD in Spain to EUR, then converts that amount in EUR to ZAR, how much will she have in South African rand? Explain the process you used to arrive at your answer.
3. Explain how to compose the functions E and R to answer Item 2. Write the composite function and identify the domain and range.
4. After converting USD to EUR in Spain, Gaby had 139 EUR. Use an inverse function to find how much USD she converted.
5. Kathryn buys a wooden bowl as a souvenir while in South Africa. She wants to ship it back to the United States. The table below shows the costs for one shipping service.

Shipping Costs to U.S.

Mass of Package (g)	Cost to Ship (ZAR)
No more than 100	14.00
More than 100 and no more than 200	28.00
More than 200 and no more than 1000	35.00

- a. Write a piecewise-defined function that gives the cost C in South African rand for shipping a package with a mass of M grams.
 - b. Write the domain of the function using an inequality, interval notation, and set notation.
 - c. Write the range of the function using set notation.
 - d. Graph the function.
 - e. The package containing Kathryn's bowl has a mass of 283 grams. If she needs to convert euros to rand to pay for the shipping, how many euros will she need? Explain how you determined your answer.
6. Kathryn and Gaby are shopping for plane tickets back to their home city of Chicago. The average cost of a plane ticket from Johannesburg, South Africa, to Chicago is \$1300. The function $g(x) = |x - 1300|$ gives the variation of a ticket costing x dollars from the average ticket price.
 - a. Graph $g(x)$.
 - b. Describe the graph of $g(x)$ as a transformation of the graph of $f(x) = |x|$.
 - c. At one travel website, all of the ticket prices are within \$200 of the average price. Explain how you can use the graph of $g(x)$ to find the least and greatest ticket prices offered at the website.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
Mathematics Knowledge and Thinking (Items 1, 3, 4, 5b, 5c, 6b)	<ul style="list-style-type: none"> Clear and accurate identification of and understanding of function concepts including domain, range, composition, inverse, and function transformations 	<ul style="list-style-type: none"> A functional understanding and accurate identification of function concepts including domain, range, composition, inverse, and function transformations 	<ul style="list-style-type: none"> Partial understanding and partially accurate identification of function concepts including domain, range, composition, inverse, and function transformations 	<ul style="list-style-type: none"> Little or no understanding and inaccurate identification of function concepts including domain, range, composition, inverse, and function transformations
Problem Solving (Items 2, 5e, 6c)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but results in a correct answer 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers 	<ul style="list-style-type: none"> No clear strategy when solving problems
Mathematical Modeling / Representations (Items 3, 4, 5a-d, 6a)	<ul style="list-style-type: none"> Fluency in creating piecewise-defined, inverse, and composite functions to model real-world scenarios Clear and accurate understanding of how to graph piecewise-defined functions and represent intervals using inequalities, interval notation, and set notation 	<ul style="list-style-type: none"> Little difficulty in creating piecewise-defined, inverse, and composite functions to model real-world scenarios Mostly accurate understanding of how to graph piecewise-defined functions and represent intervals using inequalities, interval notation, and set notation 	<ul style="list-style-type: none"> Partial understanding of how to create piecewise-defined, inverse, and composite functions to model real-world scenarios Partial understanding of how to graph piecewise-defined functions and represent intervals using inequalities, interval notation, and set notation 	<ul style="list-style-type: none"> Little or no understanding of how to create piecewise-defined, inverse, and composite functions to model real-world scenarios Inaccurate or incomplete understanding of how to graph piecewise-defined functions and represent intervals using inequalities, interval notation, and set notation
Reasoning and Communication (Items 2, 3, 5e, 6b, 6c)	<ul style="list-style-type: none"> Precise use of appropriate math terms and language to describe function transformation and function composition Clear and accurate explanation of the steps to solve a problem based on a real-world scenario 	<ul style="list-style-type: none"> Adequate description of function transformation and function composition Adequate explanation of the steps to solve a problem based on a real-world scenario 	<ul style="list-style-type: none"> Misleading or confusing description of function transformation and function composition Misleading or confusing explanation of the steps to solve a problem based on a real-world scenario 	<ul style="list-style-type: none"> Incomplete or inaccurate description of function transformation and function composition Incomplete or inadequate explanation of the steps to solve a problem based on a real-world scenario